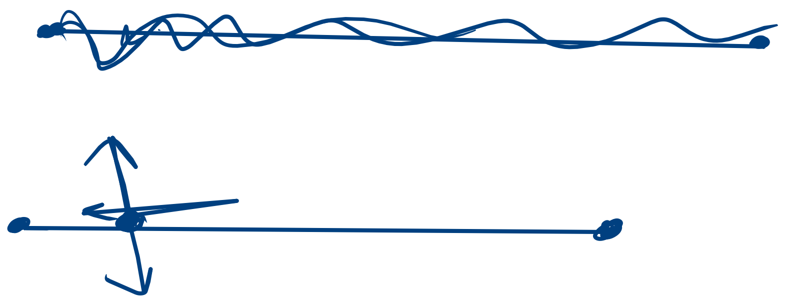
Propagation of a Disturbance



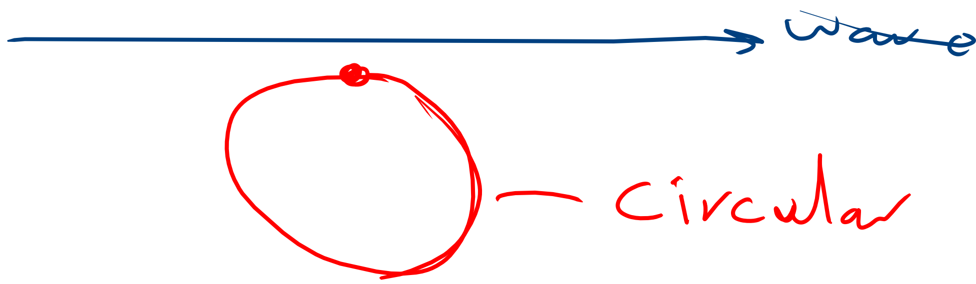
•A **wave** is a traveling disturbance that transfers **energy** from one location to another **without transporting matter**.

•In mechanical waves, **elements of the medium oscillate about the equilibrium positions** while the wave moves through the medium.

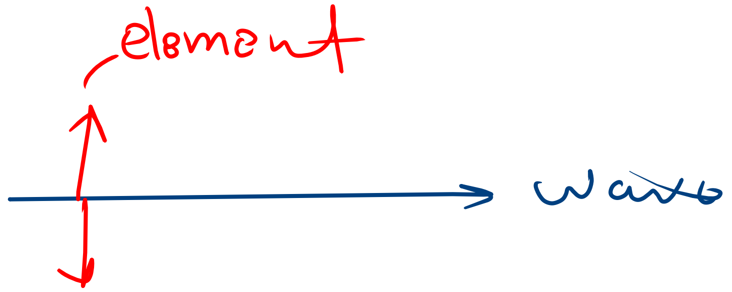
•Wave motion is produced by **restoring forces** between adjacent elements of the medium.

3) **Surface water waves** a

combination of transverse and longitudinal displacements.

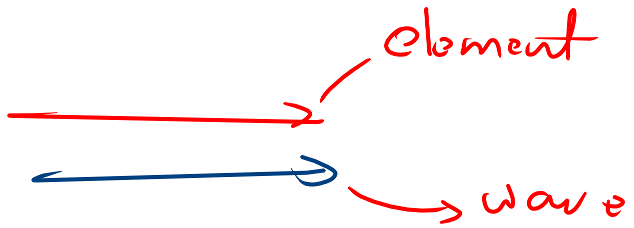


Types of Mechanical Waves



1- Transverse Waves

•In a **transverse wave**, the displacement of the medium's elements is **perpendicular** to the direction of wave propagation.



2- Longitudinal Waves

•In a **longitudinal wave**, the displacement of the medium's elements is **parallel** to the direction of wave propagation.

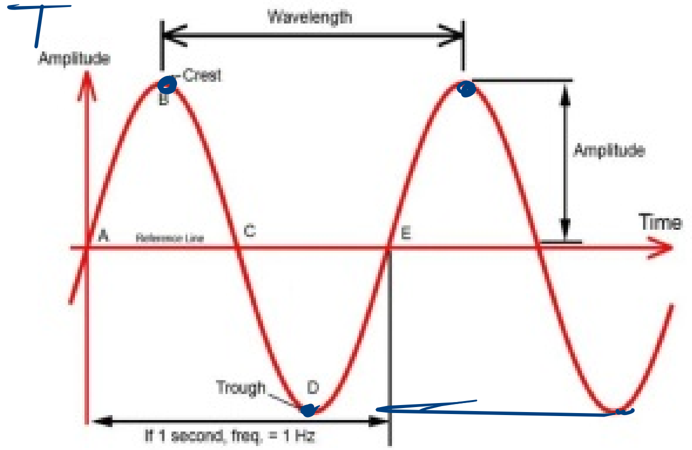
• The general mathematical form of a sinusoidal traveling wave is:

$$y = A\sin(kx - \omega t + \phi)$$

Amplitude

phase

$$\text{Frequency} = \frac{1}{T}$$



The **angular wave number** k and **angular frequency** ω **(rad/s)** are defined as

$$k = \frac{2\pi}{\lambda} \text{ rad/m} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

wavelength

period

$$\text{wave speed } (V) = \lambda F = \frac{\omega}{K}$$

The **transverse velocity** and the **transverse acceleration** of a string element are: :

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t)$$

$$V_y = \frac{dy}{dt} = -\omega A\cos(Kx - \omega t)$$

$$a_y = \frac{dV}{dt} = -\omega^2 A\sin(Kx - \omega t)$$

$$V_{max} = \omega A \quad , \quad a_{max} = \omega^2 A$$

*When **two or more traveling waves** move through the same medium, the **resultant displacement** at any point is the **algebraic sum** of the displacements due to the individual waves.*

$$y_{resultant} = y_1 + y_2 + \cdots$$

$Y_1 = Y_2$

1- ***Constructive interference*** occurs when the displacements caused by the two pulses are in the **same** direction.

The amplitude of the resultant pulse is **greater** than either individual pulse.

$Y_1 = -Y_2$

2- *Destructive interference* occurs when the displacements caused by the two pulses are **in opposite** directions.

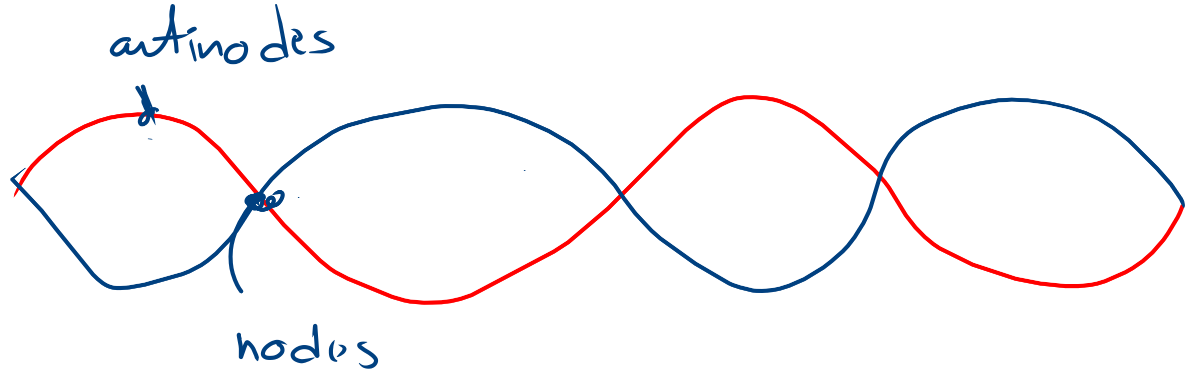
The amplitude of the resultant pulse is **less** than either individual pulse.

$$y_1 = A\sin(kx - \omega t) \quad y_2 = A\sin(kx - \omega t + \phi)$$

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$



Standing Waves

Assume two waves with the same amplitude, frequency and wavelength, traveling in opposite directions in a medium.

+ve x axis

$$y_1 = A \sin (kx - \omega t) \qquad y_2 = A \sin (kx + \omega t)$$

-ve x- axis

Amplitude

$$y = (2A \sin kx) \cos \omega t$$

The distance between adjacent antinodes is equal to $\lambda/2$.
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The distance between a node and an adjacent antinode is $\lambda/4$.

- A **node** occurs at a point of **zero amplitude**.
  - These correspond to positions of *x* where, $\sin kx = 0$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \ldots$$

◀ Positions of nodes

- An **antinode** occurs at a point of **maximum displacement, 2*A***.
  - These correspond to positions of *x* where, $\sin kx = \pm 1 \qquad kx =$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \quad n = 1, 3, 5, \ldots$$

◀ Positions of antinodes