Exercise set (4.3):

**Exercise 4.3. P.221-222:**

**1-2(a-b)-4(b)-5-6(b)-7-15-20-21-27-29-33**

**1–12** Evaluate the integrals using the indicated substitutions.

**1.** (a) $\int 2x(x^2+1)^{23}\,dx;\ u = x^2+1$

(b) $\int \cos^3 x \sin x\,dx;\ u = \cos x$

a) $u = x^2 + 1$

$du = 2x\,dx$

$\int (u)^{23}\,du$

$\frac{u^{23+1}}{23+1} + C \quad = \frac{u^{24}}{24} + C$

$= \frac{(x^2+1)^{24}}{24} + C$

b) $u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int u^3(-du) \quad = -\int u^3\,du$

$= -\frac{u^{3+1}}{3+1} + C \quad = -\frac{u^4}{4} + C$

$= -\frac{\cos^4 x}{4} + C$

**2.** (a) $\int \frac{1}{\sqrt{x}} \sin \sqrt{x}\, dx; \quad u = \sqrt{x}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$2du = \frac{1}{\sqrt{x}} dx$

$\int \sin u \,(2du) = 2\int \sin u \, du$

$-2\cos(u) + C = -2\cos(\sqrt{x}) + C$

(b) $\int \frac{3x\,dx}{\sqrt{4x^2+5}}$; $u = 4x^2 + 5$

$u = 4x^2 + 5$

$du = 8x\,dx \Rightarrow \frac{du}{8} = x\,dx$

$3\int \frac{du}{8\sqrt{u}} = \frac{3}{8}\int \frac{du}{\sqrt{u}} = \frac{3}{8} \cdot 2\sqrt{u} + C =$

$\frac{6}{8}\sqrt{4x^2+5} + C = \frac{3}{4}\sqrt{4x^2+5} + C$

**4.** (b) $\int (2x+7)(x^2+7x+3)^{4/5}\,dx;\; u = x^2+7x+3$

$u = x^2 + 7x + 3$

$du = 2x + 7\ dx$

$\int u^{4/5}\,du$

$= \dfrac{u^{4/5+1}}{4/5+1} + C$

$= \dfrac{u^{9/5}}{9/5} + C$

$= \dfrac{5}{9} u^{9/5} + C$

$= \dfrac{5}{9}(x^2+7x+3)^{9/5} + C$

**5.** (a) $\int \cot x \csc^2 x\, dx;\ u = \cot x$

(b) $\int (1+\sin t)^9 \cos t\, dt;\ u = 1 + \sin t$

a) $u = \cot x$

$du = -\csc^2 x\, dx$

$-du = \csc^2 x\, dx$

$\int u(-du) = -\int u\, du$

$= -\frac{u^2}{2} + C$

$= -\frac{\cot^2 x}{2} + C$

b) $u = 1 + \sin t$

$du = \cos t\, dt$

$\int u^9\, du = \frac{u^{10}}{10} + C$

$= \frac{(1+\sin t)^{10}}{10} + C$

**6.** (b) $\int x \sec^2 x^2 \, dx; \; u = x^2$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \sec^2 u \, \frac{du}{2} = \frac{1}{2} \int \sec^2 u \, du$

$= \frac{1}{2}(\tan u) + C$

$= \frac{1}{2} \tan x^2 + C$

**7.** (a) $\int x^2\sqrt{1+x}\,dx;\ \ u = 1 + x$

(b) $\int [\csc(\sin x)]^2 \cos x\,dx;\ \ u = \sin x$

a) $u = 1 + x$

$du = dx$

$u - 1 = x \Rightarrow (u-1)^2 = x^2$

$$\int (u-1)^2 \sqrt{u}\ du = \int (u^2 - 2u + 1)(u)^{1/2}\,du$$

$$\int u^{2+1/2} - 2u^{1+\frac{1}{2}} + u^{1/2}\ du$$

$$\int u^{5/2} - 2u^{3/2} + u^{1/2}\,du$$

$$= \frac{u^{5/2+1}}{5/2+1} - 2\frac{u^{3/2+1}}{3/2+1} + \frac{u^{1/2+1}}{1/2+1} + C$$

$$= \frac{u^{7/2}}{7/2} - 2\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{7}u^{7/2} - 2\left(\frac{2}{5}\right)u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(1+x)^{7/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C$$

b) $u = \sin x$

$du = \cos x \, dx$

$\int (\csc(u))^2 \, du$

$= -\cot u + C$

$= -\cot(\sin x) + C$

**11–36** Evaluate the integrals using appropriate substitutions.

**15.** $\int \sec 4x \tan 4x \, dx$

$u = 4x$

$du = 4\, dx \Rightarrow \frac{du}{4} = dx$

$\int \sec(u) \tan(u) \frac{du}{4}$

$\frac{1}{4} \int \sec(u) \tan(u)\, du$

$\frac{1}{4} \sec(u) + C$

$= \frac{1}{4} \sec(4x) + C$

**20.** $\displaystyle\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$

$u = x^3 + 3x$

$du = 3x^2 + 3\ dx$

$du = 3(x^2+1)\,dx$

$\dfrac{du}{3} = (x^2+1)$

$\dfrac{1}{3}\displaystyle\int \frac{du}{\sqrt{u}} \quad = \frac{1}{3}\int \frac{du}{u^{1/2}}$

$\dfrac{1}{3}\displaystyle\int u^{-1/2}\,du \ = \frac{1}{3}\cdot\frac{u^{-1/2+1}}{-1/2+1} + C$

$= \dfrac{1}{3}\cdot\dfrac{u^{1/2}}{\frac{1}{2}} + C$

$= \dfrac{1}{3}\cdot\dfrac{2}{1}\cdot u^{1/2} + C$

$= \dfrac{2}{3}\sqrt{u} + C$

$= \dfrac{2}{3}\sqrt{x^3+3x} + C$

**21.** $\int \frac{x^3}{(5x^4+2)^3}\,dx$

$u=(5x^4+2)$

$du=20x^3\,dx$

$\frac{du}{20}=x^3\,dx$

$\int \frac{1}{u^3}\cdot\frac{du}{20}$

$\frac{1}{20}\int u^{-3}\,du=\frac{1}{20}\cdot\frac{u^{-3+1}}{-3+1}+C$

$=\frac{1}{20}\cdot\frac{u^{-2}}{-2}+C$

$=-\frac{1}{40}\cdot\frac{1}{u^2}+C$

$=-\frac{1}{40}\cdot\frac{1}{(5x^4+2)^2}+C$

$=-\frac{1}{40(5x^4+2)^2}+C$

**27.** $\int x \sec^2(x^2)\, dx$

$u = x^2$

$du = 2x\, dx \Rightarrow \frac{du}{2} = x\, dx$

$\frac{1}{2}\int \sec^2(u)\, du$

$= \frac{1}{2}\tan(u) + C$

$= \frac{1}{2}\tan(x^2) + C$

**29.** $\int \cos 4\theta \sqrt{2 - \sin 4\theta}\, d\theta$

$u = 2 - \sin 4\theta$

$du = -\cos 4\theta \cdot 4\, d\theta$

$-\frac{du}{4} = \cos(4\theta)\, d\theta$

$\int -\frac{1}{4} \sqrt{u}\, du$

$= -\frac{1}{4} \int u^{1/2}\, du = -\frac{1}{4} \cdot \frac{u^{1/2+1}}{1/2+1} + C$

$= -\frac{1}{4} \frac{u^{3/2}}{3/2} + C$

$= -\frac{2}{12} u^{3/2} + C$

$= -\frac{1}{6} (2 - \sin 4\theta)^{3/2} + C$

**33.** $\displaystyle\int \frac{y}{\sqrt{2y+1}}\,dy$

$u = 2y + 1$

$du = 2\,dy \Rightarrow \frac{du}{2} = dy$

$u - 1 = 2y \Rightarrow y = \frac{u-1}{2}$

$$\int \frac{u-1}{2\sqrt{u}} \cdot \frac{du}{2}$$

$$= \frac{1}{4}\int \frac{u-1}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du$$

$$= \frac{1}{4}\int u^{1/2} - u^{-1/2}\,du$$

$$= \frac{1}{4}\left(\frac{u^{1/2+1}}{1/2+1} - \frac{u^{-1/2+1}}{-1/2+1}\right) + C$$

$$= \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + C$$

$$= \frac{2}{12}u^{3/2} - \frac{2}{4}u^{1/2} + C$$

$$= \frac{1}{6}u^{3/2} - \frac{1}{2}u^{1/2} + C$$

$$= \frac{1}{6}(2y+1)^{3/2} - \frac{1}{2}(2y+1)^{1/2} + C$$