Exercise set (4.2):

Exercise set 4.2:

1, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 29, 30, 33. p. 215

EXERCISE SET 4.2 Graphing Utility C CAS

1. In each part, confirm that the formula is correct, and state a corresponding integration formula.

(a) $\frac{d}{dx}[\sqrt{1+x^2}\,] = \frac{x}{\sqrt{1+x^2}}$

(b) $\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2\cos(1+x^3)$

a) $\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{1}{2\sqrt{1+x^2}} \cdot 2x$

$= \frac{x}{\sqrt{1+x^2}}$

$\therefore$ formula is Correct

$\int \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2} + C$

b) $\frac{d}{dx}\left(\frac{1}{3}\sin(1+x^3)\right) = \frac{1}{3}\cos(1+x^3).(3x^2)$

$= x^2\cos(1+x^3)$

$\therefore$ formula is Correct

$\int x^2\cos(1+x^3)\,dx = \frac{1}{3}\sin(1+x^3) + C$

5–8 Find the derivative and state a corresponding integration formula.

5. $\dfrac{d}{dx}[\sqrt{x^3+5}\,]$

$$= \frac{3x^2}{2\sqrt{x^3+5}}$$

$$\int \frac{3x^2}{2\sqrt{x^3+5}}\,dx = \sqrt{x^3+5} + C$$

8. $\frac{d}{dx}[\sin x - x\cos x]$

$= \cos x - 1.\cos x - x(-\sin x)$

$= \cancel{\cos x} - \cancel{\cos x} + x\sin x$

$= x\sin x$

$\int x\sin x\,dx = \sin x - x\cos x + C$

9–10 Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule (Formula 2 in Table 4.2.1). ■

(a) $\int x^8\,dx$

$= \frac{x^9}{9} + C$

(b) $\int x^{5/7}\,dx$

$= \frac{x^{12/7}}{12/7} + C$

$= \frac{7}{12}x^{12/7} + C$

(c) $\int x^3\sqrt{x}\,dx$

$\int x^3 x^{1/2}\,dx = \int x^{3+\frac{1}{2}}\,dx$

$= \int x^{7/2}\,dx$

$= \frac{x^{7/2+1}}{7/2+1} + C$

$= \frac{x^{9/2}}{9/2} + C = \frac{2}{9}x^{9/2} + C$

10. (a) $\int \sqrt[3]{x^2}\, dx$

$= \int (x^{1/3})^2\, dx$

$= \int x^{2/3}\, dx$

$= \dfrac{x^{5/3}}{5/3} + C$

$= \dfrac{3}{5} x^{5/3} + C$

(b) $\int \dfrac{1}{x^6}\, dx$

$= \int x^{-6}\, dx$

$= -\dfrac{x^{-5}}{5} + C$

(c) $\int x^{-7/8}\, dx$

$= \dfrac{x^{-7/8+1}}{-7/8+1} + C$

$= \dfrac{x^{1/8}}{1/8} + C$

$= 8x^{1/8} + C$

$= 8\sqrt[8]{x} + C$

11–14 Evaluate each integral by applying Theorem 4.2.3 and Formula 2 in Table 4.2.1 appropriately. ■

11. $\int \left[5x + \frac{2}{3x^5}\right] dx$

$$\int 5x\,dx + \int \frac{2}{3x^5}\,dx$$

$$5\int x\,dx + \frac{2}{3}\int \frac{1}{x^5}\,dx$$

$$5\int x\,dx + \frac{2}{3}\int x^{-5}\,dx$$

$$5\frac{x^{1+1}}{1+1} + \frac{2}{3}\left(\frac{x^{-5+1}}{-5+1}\right) + C$$

$$5\frac{x^2}{2} + \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + C$$

$$\frac{5x^2}{2} - \frac{2x^{-4}}{12} + C$$

$$\frac{5x^2}{2} - \frac{x^{-4}}{6} + C$$

$$\frac{5x^2}{2} - \frac{1}{6x^4} + C$$

12. $\int [x^{-1/2} - 3x^{7/5} + \frac{1}{9}]\,dx$

$\int x^{-1/2} dx - 3\int x^{7/5} dx + \int \frac{1}{9} dx$

$= \frac{x^{1/2}}{1/2} - 3\frac{x^{12/5}}{12/5} + \frac{1}{9}x + C$

$= 2x^{1/2} - 3\frac{5}{12}x^{12/5} + \frac{1}{9}x + C$

$= 2x^{1/2} - \frac{5}{4}x^{12/5} + \frac{1}{9}x + C$

13. $\int [x^{-3} - 3x^{1/4} + 8x^2]\,dx$

$= \frac{x^{-2}}{-2} - 3\frac{x^{5/4}}{5/4} + 8\frac{x^3}{3} + C$

$= -\frac{x^{-2}}{2} - \frac{12}{5}x^{5/4} + 8\frac{x^3}{3} + C$

14. $\int \left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}} \right] dy$

$= 10 \int y^{-3/4} \, dy - \int y^{1/3} \, dy + 4 \int y^{-1/2} \, dy$

$= 10 \, \frac{y^{1/4}}{1/4} - \frac{y^{4/3}}{4/3} + 4 \, \frac{y^{1/2}}{1/2} + C$

$= 40 y^{1/4} - \frac{3}{4} y^{4/3} + 8 y^{1/2} + C$

$= 40 \sqrt[4]{y} - \frac{3}{4} \sqrt[3]{y^4} + 8 \sqrt{y} + C$

15–30 Evaluate the integral and check your answer by differentiating.

15. $\int x(1+x^3)\,dx$

$= \int x + x^4 \, dx$

$= \frac{x^2}{2} + \frac{x^5}{5} + C$

_ Check the answer by differentiating

$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5} + C\right)$

$= \frac{\cancel{2}x}{\cancel{2}} + \frac{5x^4}{5}$

$= x + x^4$

16. $\int (2+y^2)^2\,dy$

$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

$\int (4 + 4y^2 + y^4)\,dy$

$\int 4\,dy + 4\int y^2\,dy + \int y^4\,dy$

$= 4y + 4\dfrac{y^3}{3} + \dfrac{y^5}{5}$

_ Check the answer by differentiating

$\dfrac{d}{dy}\left(4y + 4\dfrac{y^3}{3} + \dfrac{y^5}{5}\right)$

$= 4 + \cancel{3}\left(\dfrac{4}{\cancel{3}}\right)y^{3-1} + \cancel{5}\left(\dfrac{1}{\cancel{5}}\right)y^{5-1}$

$= 4 + 4y^2 + y^4$

$= (2+y^2)^2$

17. $\int x^{1/3}(2-x)^2\,dx$

$$=\int x^{1/3}(4-4x+x^2)\,dx$$

$$=\int (4x^{1/3}-4x^{4/3}+x^{7/3})\,dx$$

$$=4\,\frac{x^{4/3}}{4/3}-4\,\frac{x^{7/3}}{7/3}+\frac{x^{10/3}}{10/3}+C$$

$$=3x^{4/3}-\frac{12}{7}x^{7/3}+\frac{3}{10}x^{10/3}+C$$

- Check the answer by differentiating

$$\frac{d}{dx}\left(3x^{4/3}-\frac{12}{7}x^{7/3}+\frac{3}{10}x^{10/3}+C\right)$$

$$=4x^{1/3}-4x^{4/3}+x^{7/3}$$

$$=x^{1/3}(4-4x+x^2)$$

$$=x^{1/3}(2-x)^2$$

18. $\int (1 + x^2)(2 - x)\, dx$

$$\int (2 - x + 2x^2 - x^3)\, dx$$

$$= 2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C$$

_ Check the answer by differentiating

$$\frac{d}{dx}\left(2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C\right)$$

$$= 2 - x + 2x^2 - x^3$$

19. $\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx$

$$\int X^{-4}(X^5 + 2X^2 - 1)\,dX$$

$$\int (X + 2X^{-2} - X^{-4})\,dX$$

$$= \frac{X^2}{2} + 2\,\frac{X^{-1}}{-1} - \frac{X^{-3}}{-3} + C$$

$$= \frac{X^2}{2} - \frac{2}{X^1} + \frac{1}{3X^3} + C$$

_ Check the answer by differentiating

$$\frac{d}{dX}\left(\frac{X^2}{2} - \frac{2}{X^1} + \frac{1}{3X^3} + C\right)$$

$$= X + 2X^{-2} - X^{-4})$$

20. $\int \frac{1-2t^3}{t^3}\,dt$

$\int t^{-3}(1-2t^3)\,dt$

$\int (t^{-3}-2)\,dt$

$= \frac{t^{-2}}{-2} - 2t + C$

$= \frac{1}{2t^2} - 2t + C$

- Check the answer by differentiating

$\frac{d}{dt}\left(\frac{1}{2t^2} - 2t + C\right)$

$= t^{-3} - 2$

23. $\int \sec x(\sec x + \tan x)\,dx$

$\int (\sec^2 x + \sec x \tan x)dx$

$= \tan x + \sec x + C$

24. $\int \csc x(\sin x + \cot x)\,dx$

$\int (\csc x \sin x + \csc x \cot x)dx$

$\int (\frac{1}{\sin x} \cdot \sin x + \csc x \cot x)dx$

$\int (1 + \csc x \cot x)dx$

$= x - \csc x + C$

25. $\int \frac{\sec\theta}{\cos\theta}\,d\theta$

$$\boxed{\frac{1}{\cos\theta} = \sec\theta}$$

$$\int \sec\theta \cdot \frac{1}{\cos\theta}\,d\theta$$

$$\int \sec\theta \cdot \sec\theta\,d\theta$$

$$\int \sec^2\theta\,d\theta$$

$$= \tan\theta + C$$

_ Check the answer by differentiating

$$\frac{d}{d\theta}\left(\tan\theta + C\right)$$

$$= \sec^2\theta$$

$$= \sec\theta \cdot \sec\theta$$

$$= \sec\theta \cdot \frac{1}{\cos\theta}$$

$$= \frac{\sec\theta}{\cos\theta}$$

26. $\int \frac{dy}{\csc y}$

$= \int \frac{1}{\csc y} \, dy$

$= \int \sin y \, dy$

$= \cos y + C$

27. $\int \frac{\sin x}{\cos^2 x} \, dx$

$= \int \frac{\sin x}{\cos x \cos x} \, dx$

$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx$

$= \int \tan x \sec x \, dx$

$= \sec x + C$

29. $\int [1 + \sin^2 \theta \csc \theta] \, d\theta$

$$= \int \left(1 + \sin^{\cancel{2}} \theta \cdot \frac{1}{\cancel{\sin \theta}} \right) d\theta$$

$$= \int \left(1 + \sin \theta \right) d\theta$$

$$= \theta - \cos \theta + C$$

30. $\int \frac{\sec x + \cos x}{2 \cos x} \, dx$

$$= \frac{1}{2} \int \left(\frac{\sec x}{\cos x} + \frac{\cos x}{\cos x} \right) dx$$

$$= \frac{1}{2} \int \left(\frac{\frac{1}{\cos x}}{\cos x} + 1 \right) dx$$

$$= \frac{1}{2} \int \left(\frac{1}{\cos^2 x} + 1 \right) dx$$

$$= \frac{1}{2} \tan x + x + C$$

33–36 True–False Determine whether the statement is true or false. Explain your answer.

33. If $F(x)$ is an antiderivative of $f(x)$, then (true)

$$\int f(x)\,dx = F(x) + C$$