Exercise set (4.2):

1(b)-2(a)-5-6-
9(c)-10(b)-13-14-
16-18-19-24-27

EXERCISE SET 4.2

Graphing Utility   C CAS

1. In each part, confirm that the formula is correct, and state a corresponding integration formula.

(a) $\frac{d}{dx}[\sqrt{1+x^2}] = \frac{x}{\sqrt{1+x^2}}$

(b) $\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2\cos(1+x^3)$

b) $\frac{d}{dx}\left(\frac{1}{3}\sin(1+x^3)\right) = \frac{1}{3}\cos(1+x^3).(3x^2)$

$= x^2\cos(1+x^3)$

$\therefore$ formula is Correct

$\int x^2\cos(1+x^3)\,dx = \frac{1}{3}\sin(1+x^3) + C$

2. In each part, confirm that the stated formula is correct by differentiating.

(a) $\int x \sin x \, dx = \sin x - x \cos x + C$

$$\frac{d}{dx}(\sin x - x\cos x + C) =$$

$$= \cos x - \left(\frac{d}{dx}(x).(\cos x) + (x).\frac{d}{dx}(\cos x)\right)$$

$$= \cos x - \cos x + x(-\sin x)$$

$$= \cancel{\cos x - \cos x} + x\sin x$$

$$= x\sin x$$

$\therefore$ formula is Correct

5–8 Find the derivative and state a corresponding integration formula.

5. $\frac{d}{dx}[\sqrt{x^3+5}]$

$= \frac{3x^2}{2\sqrt{x^3+5}}$

$\int \frac{3x^2}{2\sqrt{x^3+5}}\,dx = \sqrt{x^3+5} + C$

6. $\frac{d}{dx}\left[\frac{x}{x^2+3}\right]$

$= \frac{(1)(x^2+3)-(x)(2x)}{(x^2+3)^2}$

$= \frac{(x^2+3)-2x^2}{(x^2+3)^2}$

$= \frac{-x^2+3}{(x^2+3)^2}$

$\int \frac{-x^2+3}{(x^2+3)^2}\,dx = \frac{x}{(x^2+3)} + C$

9–10 Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule (Formula 2 in Table 4.2.1).

(c) $\int x^3\sqrt{x}\,dx$

$\int x^3 x^{1/2}\,dx = \int x^{3+\frac{1}{2}}\,dx$

$= \int x^{7/2}\,dx$

$= \dfrac{x^{7/2+1}}{7/2+1} + C$

$= \dfrac{x^{9/2}}{9/2} + C = \dfrac{2}{9}x^{9/2} + C$

10. (b) $\int \frac{1}{x^6}\,dx$

$= \int x^{-6}\,dx$

$= -\dfrac{x^{-5}}{5} + C$

11–14 Evaluate each integral by applying Theorem 4.2.3 and Formula 2 in Table 4.2.1 appropriately.

13. $\int [x^{-3} - 3x^{1/4} + 8x^2]\, dx$

$= \frac{x^{-2}}{-2} - 3\frac{x^{5/4}}{5/4} + 8\frac{x^3}{3} + C$

$= -\frac{x^{-2}}{2} - \frac{12}{5} x^{5/4} + 8\frac{x^3}{3} + C$

14. $\int \left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}}\right] dy$

$= 10\int y^{-3/4}\, dy - \int y^{1/3}\, dy + 4\int y^{-1/2}\, dy$

$= 10\frac{y^{1/4}}{1/4} - \frac{y^{4/3}}{4/3} + 4\frac{y^{1/2}}{1/2} + C$

$= 40y^{1/4} - \frac{3}{4}y^{4/3} + 8y^{1/2} + C$

$= 40\sqrt[4]{y} - \frac{3}{4}\sqrt[3]{y^4} + 8\sqrt{y} + C$

15–30 Evaluate the integral and check your answer by differentiating.

16. $\int (2+y^2)^2\, dy$

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

$\int (4 + 4y^2 + y^4)\, dy$

$\int 4\, dy + 4\int y^2\, dy + \int y^4\, dy$

$= 4y + 4\dfrac{y^3}{3} + \dfrac{y^5}{5}$

– Check the answer by differentiating

$\dfrac{d}{dy}\left(4y + 4\dfrac{y^3}{3} + \dfrac{y^5}{5}\right)$

$= 4 + \cancel{3}\left(\dfrac{4}{\cancel{3}}\right) y^{3-1} + \cancel{5}\left(\dfrac{1}{\cancel{5}}\right) y^{5-1}$

$= 4 + 4y^2 + y^4$

$= (2+y^2)^2$

18. $\int (1 + x^2)(2 - x)\, dx$

$\int (2 - x + 2x^2 - x^3)\, dx$

$= 2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C$

_ Check the answer by differentiating

$\frac{d}{dx}\left(2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C\right)$

$= 2 - x + 2x^2 - x^3$

19. $\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$

$\int x^{-4}(x^5 + 2x^2 - 1)\, dx$

$\int (x + 2x^{-2} - x^{-4})\, dx$

$= \frac{x^2}{2} + 2\frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C$

$= \frac{x^2}{2} - \frac{2}{x^1} + \frac{1}{3x^3} + C$

_ Check the answer by differentiating

$\frac{d}{dx}\left(\frac{x^2}{2} - \frac{2}{x^1} + \frac{1}{3x^3} + C\right)$

$= x + 2x^{-2} - x^{-4})$

24. $\int \csc x(\sin x + \cot x)\, dx$

$\int(\csc x \sin x + \csc x \cot x)dx$

$\int(\frac{1}{\sin x} \cdot \sin x + \csc x \cot x)dx$

$\int(1 + \csc x \cot x)dx$

$= x - \csc x + C$

27. $\int \frac{\sin x}{\cos^2 x}\,dx$

$$= \int \frac{\sin x}{\cos x \cos x}\,dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\,dx$$

$$= \int \tan x \sec x\,dx$$

$$= \sec x + C$$