جامعة الأميرة نورة بنت عبدالرحمن

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Exercise set (0.1)

Exercise set 0.1:

1 from quick check exercises p.11

3, 4, 7, 8 (a), 9 (a,b,c), 10 (b, d, e), (15, 16, 17, 18 without the part of determining a formula for y in term of x) 19, 20, 21, 22 p.12-13.

✔ QUICK CHECK EXERCISES 0.1 (See page 15 for answers.)

1. Let $f(x) = \sqrt{x+1} + 4$.

(a) The natural domain of f is ________.

(b) $f(3) =$ ________

(c) $f(t^2 - 1) =$ ________

(d) $f(x) = 7$ if $x =$ ________

(e) The range of f is ________.

a) $x + 1 \geqslant 0$ مجال الدالة الجذرية ماتحت الجذر أكبر من او يساوي الصفر اذا كان الجذر في البسط

$x \geqslant -1$ ننقل الواحد الى الطرف الثاني بعكس اشارته

$D = [-1, +\infty)$ يكون القوس مغلق من جهة السالب واحد بسبب أكبر من او

b) $f(3) = \sqrt{3+1} + 4$ تعويض مباشر قيمة x ب ثلاثة

$= \sqrt{4} + 4 = 2 + 4 = 6$

c) $f(t^2 - 1) = \sqrt{(t^2 - 1) + 1} + 4$

$= \sqrt{t^2} + 4 = t + 4$

d) $f(x) = 7$ if $x =$

$x = 8 \longrightarrow f(8) = \sqrt{8+1} + 4$

$= \sqrt{9} + 4 = 3 + 4 = 7$

e) The range of is

$$f(x) = \sqrt{x+1} + 4$$

$$x + 1 \geqslant 0$$

$$\sqrt{x+1} \geqslant \sqrt{0}$$

$$\sqrt{x+1} + 4 \geqslant 4$$

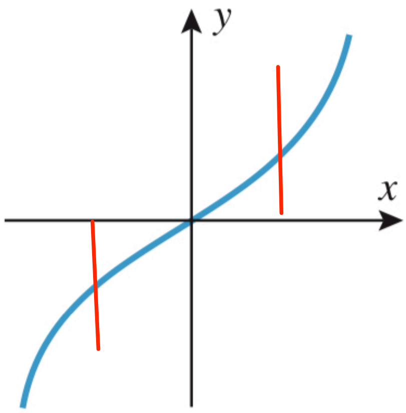
$$R = [4, +\infty)$$

EXERCISE SET 0.1

Graphing Utility

3. In each part of the accompanying figure, determine whether the graph defines y as a function of x.

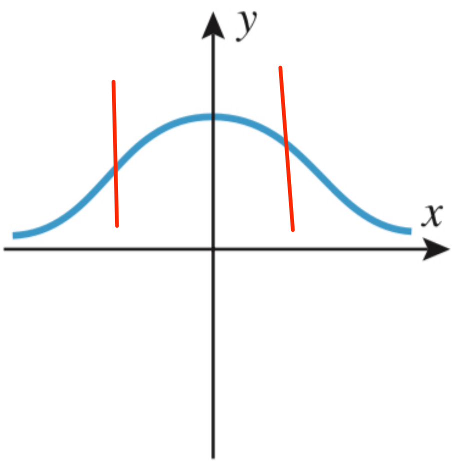
هنا نختبر هل الرسمة دالة او لا عن طريق اختبار الخط
العمودي إذا قطع الخط العمودي الدالة في أكثر من نقطة
واحدة اذاً هذه ليست دالة واذا قطع مرة واحدة فهي دالة



(a)

a) Y is function of X

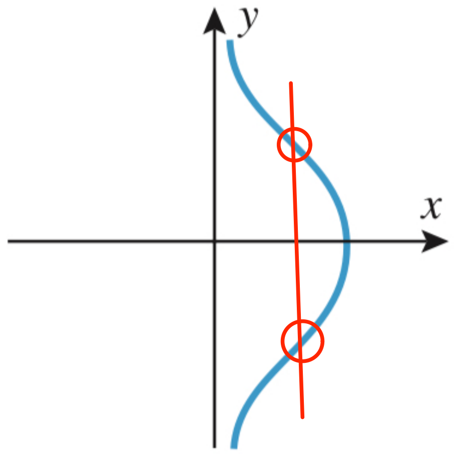
by vertical line test



(b)

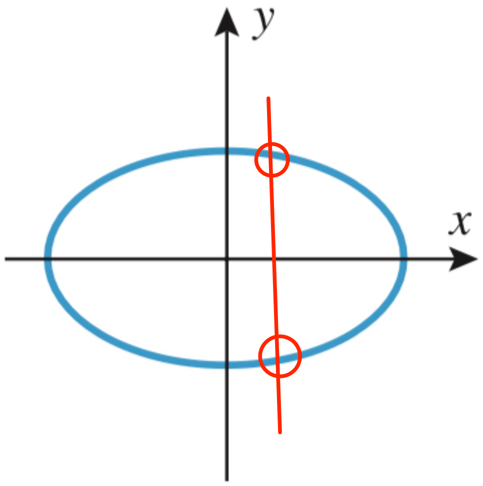
b) Y is function of X

by vertical line test



(c)

c) Y is not function of X
by vertical line test



(d)

d) Y is not function of X
by vertical line test

4. In each part, compare the natural domains of f and g.

(a) $f(x) = \dfrac{x^2 + x}{x + 1}$; $g(x) = x$

for $f(X) = \dfrac{a}{b} \longrightarrow b \neq 0$

$X + 1 = 0$

$X = -1$

Domain:

$D_f = \{X : X = -1\} = (-\infty, -1) \cup (-1, +\infty)$

$D_g = \{R\} = (-\infty, +\infty)$

$f(X) = \dfrac{X\cancel{(X+1)}}{\cancel{(X+1)}} = X$

$f(X) = g(X)$

(b) $f(x) = \dfrac{x\sqrt{x} + \sqrt{x}}{x+1}$; $g(x) = \sqrt{x}$

for $f(X) = \dfrac{a}{b} \longrightarrow b \neq 0$

$X + 1 = 0$

$X = -1$

Domain:

$D_f = X \geqslant 0 = [0, +\infty)$

$D_g = X \geqslant 0 = [0, +\infty)$

$$f(X) = \frac{X\sqrt{X} + \sqrt{X}}{X+1} = \frac{\sqrt{X}\,\cancel{(X+1)}}{\cancel{(X+1)}} = \sqrt{X}$$

$f(X) = g(X)$

7. Find $f(0)$, $f(2)$, $f(-2)$, $f(3)$, $f(\sqrt{2})$, and $f(3t)$.

(a) $f(x) = 3x^2 - 2$

$f(0) = 3(0)^2 - 2 = -2$

$f(2) = 3(2)^2 - 2 = 12 - 2 = 10$

$f(-2) = 3(-2)^2 - 2 = 12 - 2 = 10$

$f(3) = 3(3)^2 - 2 = 27 - 2 = 25$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 6 - 2 = 4$

$f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$

(b) $f(x) = \begin{cases} \dfrac{1}{x}, & x > 3 \\ 2x, & x \leq 3 \end{cases}$

$f(0) = 2(0) = 0$

$f(2) = 2(2) = 4$

$f(-2) = 2(-2) = -4$

$f(3) = 2(3) = 6$

$f(\sqrt{2}) = 2\sqrt{2}$

$$f(3t) = \begin{cases} \dfrac{1}{3t} & 3t > 3 \\ 2(3t) & 3t \leq 3 \end{cases}$$

$$= \begin{cases} \dfrac{1}{3t} & \dfrac{3t}{3} > \dfrac{3}{3} \\ 6t & \dfrac{3t}{3} \leq \dfrac{3}{3} \end{cases}$$

$$= \begin{cases} \dfrac{1}{3t} & t > 1 \\ 6t & t \leq 1 \end{cases}$$

8. Find $g(3)$, $g(-1)$, $g(\pi)$, $g(-1.1)$, and $g(t^2 - 1)$.

(a) $g(x) = \dfrac{x+1}{x-1}$

$$g(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

$$g(-1) = \frac{-1+1}{-1-1} = \frac{0}{-2} = 0$$

$$g(\pi) = \frac{\pi+1}{\pi-1}$$

$$g(-1.1) = \frac{-1.1+1}{-1.1-1} = \frac{-0.1}{-2.1} = \frac{0.1}{2.1}$$

$$g(t^2-1) = \frac{t^2-1+1}{t^2-1-1} = \frac{t^2}{t^2-2}$$

9–10 Find the natural domain and determine the range of each function. If you have a graphing utility, use it to confirm that your result is consistent with the graph produced by your graphing utility. [*Note:* Set your graphing utility in radian mode when graphing trigonometric functions.]

9. (a) $f(x) = \dfrac{1}{x-3}$

مجال الدالة الكسرية نستبعد القيم التي تسبب ان المقام يساوي صفر

$X - 3 = 0$

نساوي المقام بالصفر لنستبعد القيمة التي تسبب ان المقام يساوي صفر

$X = 3$

نقلنا 3 - للطرف الاخر فاصبحت موجبة

القيمة التي استبعدناها من المجال هي 3 لانها تسبب ان المقام يساوي صفر

$D = R - \{3\} = (-\infty, 3) \cup (3, +\infty)$

نكتب المجال بهذه الطريقة والاتحاد يكون لاستبعاد القيمة الغير موجودة في المجال

Range:

نوجد المدى للدالة الكسرية عن طريق ايجاد معكوس الدالة

$f(X) = \dfrac{1}{X-3}$

نبدل f(x) ب y

$Y = \dfrac{1}{X-3}$

ضربنا في (x-3) للطرفين لايجاد معادلة x

$Y(X-3) = 1$

$YX - 3Y = 1$

$YX = 1 + 3Y$

$X = \dfrac{1+3Y}{Y}$

بدلنا y ب x

$f^{-1}(X) = \dfrac{1+3X}{X}$

مجال الدالة العكسية هو مدى الدالة الاصلية لذالك اوجدنا معكوس الدالة

$D_{f^{-1}} = (-\infty, 0) \cup (0, +\infty) = R_f$

(b) $F(x) = \dfrac{x}{|x|}$

$x = 0$

$D = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

Range:

$x > 0 \longrightarrow f(x) = \dfrac{x}{|x|} = \dfrac{x}{x} = 1$

$x < 0 \longrightarrow f(x) = \dfrac{x}{|x|} = \dfrac{x}{-x} = -1$

$R = \{-1, 1\}$

(c) $g(x) = \sqrt{x^2 - 3}$

Domain:

$x^2 - 3 \geqslant 0 \longrightarrow x^2 \geqslant 3$

$\sqrt{x^2} \geqslant \sqrt{3} \longrightarrow |x| \geqslant \sqrt{3}$

$|a| \geqslant b \longrightarrow a \geqslant b$ or $a \leqslant -b$

$x \geqslant \sqrt{3}$ or $x \leqslant -\sqrt{3}$

$D_f = (-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$

Range:

$x^2 - 3 \geqslant 0$

$\sqrt{x^2 - 3} \geqslant \sqrt{0}$

$\sqrt{x^2 - 3} \geqslant 0$

$R = [0, +\infty)$

10. (b) $F(x) = \sqrt{4 - x^2}$

Domain:

$4 - x^2 \geqslant 0$

$x^2 \leqslant 4$

$\sqrt{x^2} \leqslant \sqrt{4} \longrightarrow |x| \leqslant 2$

$|a| \leqslant b \longrightarrow -b \leqslant a \leqslant b$

$|x| \leqslant 2 \longrightarrow -2 \leqslant x \leqslant 2$

$D = [-2, 2]$

Range:

$-2 \leqslant x \leqslant 2$

$0 \leqslant x^2 \leqslant 4$

$0 \geqslant -x^2 \geqslant -4$

$4 \geqslant 4 - x^2 \geqslant 0$

$\sqrt{4} \geqslant \sqrt{4 - x^2} \geqslant \sqrt{0}$

$2 \geqslant \sqrt{4 - x^2} \geqslant 0$

$R = [0, 2]$

d) $G(X) = X^3 + 2$

$D = R = (-\infty, +\infty)$

$R = R = (-\infty, +\infty)$

e) $h(X) = 3\sin X$

Domain:

$D = R = (-\infty, +\infty)$

Range:

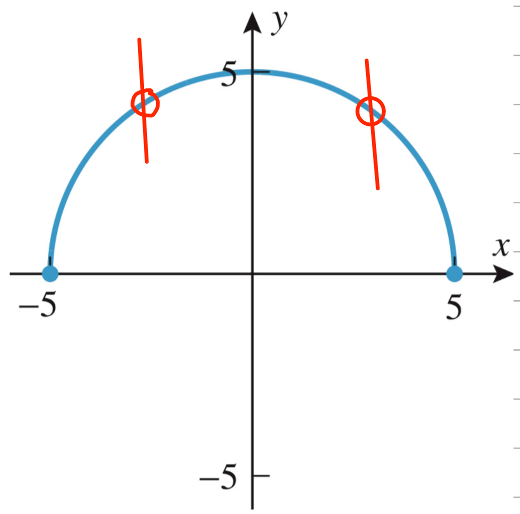
$-1 \leqslant \sin X \leqslant 1$

$-3 \leqslant 3\sin X \leqslant 3$

$R = [-3, 3]$

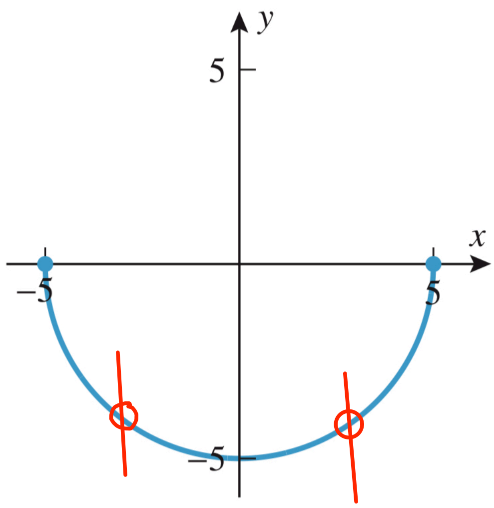
15–18 As seen in Example 3, the equation $x^2 + y^2 = 25$ does not define y as a function of x. Each graph in these exercises is a portion of the circle $x^2 + y^2 = 25$. In each case, determine whether the graph defines y as a function of x, and if so, give a formula for y in terms of x. ■

15.



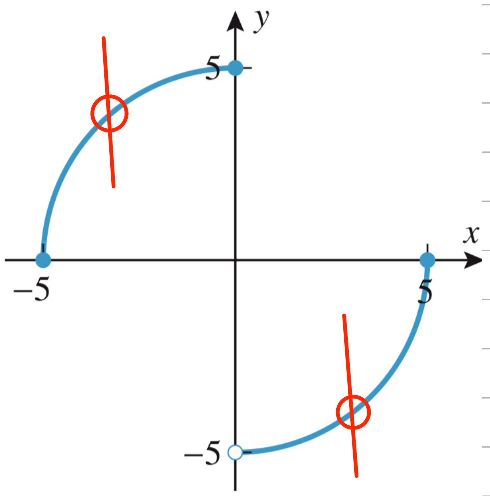
Yes, y is function of x

16.



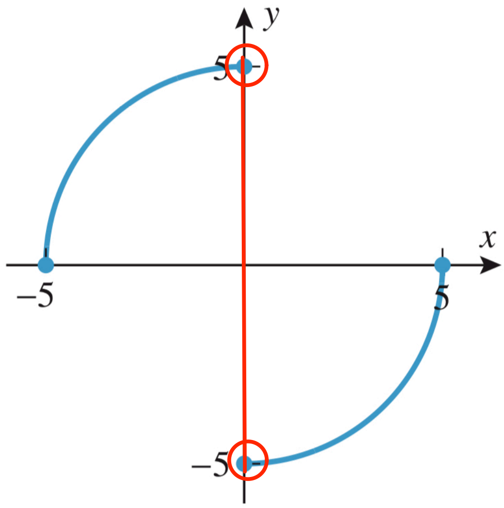
Yes, y is function of x

17.



Yes, y is function of x

18.



No, y is not function of x

19–22 True–False Determine whether the statement is true or false. Explain your answer.

19. A curve that crosses the x-axis at two different points cannot be the graph of a function. (f)

The graph of X^2-1 cross the X-axis at $X=1$ and $X=-1$

20. The natural domain of a real-valued function defined by a formula consists of all those real numbers for which the formula yields a real value. (T)

21. The range of the absolute value function is all positive real numbers. (f)

The range is also include zero

22. If $g(x) = 1/\sqrt{f(x)}$, then the domain of g consists of all those real numbers x for which $f(x) \neq 0$. (f)

$f(X) > 0$