IQ ACADEMY

Exercise set (2.2)

1. Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

a) $A \cap B$
b) $A \cup B$
c) $A - B$
d) $B - A$

A = Student who live within one mile of Student

B = Student who walk to Classes

a) $A \cap B$ = Student who both live within one mile and walk to Classes

b) $A \cup B$ = Student who live within one mile or walk to Classes

c) $A - B$ = Student who live within one mile but not walk to Classes

d) $B - A$ = Student who walk to Classes but not live within one mile

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3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a) $A \cup B$.

b) $A \cap B$.

c) $A - B$.

d) $B - A$.

a) $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

b) $A \cap B = \{3\}$

c) $A - B = \{1, 2, 4, 5\}$

d) $B - A = \{0, 6\}$

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$.
b) $A \cap B$.
c) $A - B$.
d) $B - A$.

a) $A \cup B = \{a, b, c, d, e, f, g, h\}$

b) $A \cap B = \{a, b, c, d, e\}$

c) $A - B = \emptyset$

d) $B - A = \{f, g, h\}$

11. Let A and B be sets. Prove the commutative laws from Table 1 by showing that

a) $A \cup B = B \cup A$.

b) $A \cap B = B \cap A$.

$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws

a) $A \cup B = \{x \mid x \in A \vee x \in B\}$

$= \{x \mid x \in B \vee x \in A\}$

$= B \cup A$

b) $A \cap B = \{x \mid x \in A \wedge x \in B\}$

$= \{x \mid x \in B \wedge x \in A\}$

$= B \cap A$

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14. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

$A = (A - B) \cup (A \cap B)$

$A = \{1, 5, 7, 8\} \cup \{3, 6, 9\}$

$= \{1, 3, 5, 6, 7, 8, 9\}$

$B = (B - A) \cup (A \cap B)$

$= \{2, 10\} \cup \{3, 6, 9\}$

$= \{2, 3, 6, 9, 10\}$

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16. Let A and B be sets. Show that

a) $(A \cap B) \subseteq A$.

b) $A \subseteq (A \cup B)$.

c) $A - B \subseteq A$.

d) $A \cap (B - A) = \emptyset$.

e) $A \cup (B - A) = A \cup B$.

a) let $x \in (A \cap B)$

$x \in A$ and $x \in B$

$x \in A$

b) $A \subseteq (A \cup B)$

let $x \in A$

$x \in A$ or $x \in B$

$x \in (A \cup B)$

c) $A - B \subseteq A$

let $x \in (A - B)$

$x \in A$ and $x \notin B$

$x \in A$

d) $A \cap (B - A) = \emptyset$

$A - B = A \cap \overline{B}$

$A \cap (B - A) = A \cap (B \cap \overline{A})$

$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws

$(A \cap \overline{A}) \cap B$ (associative law)

$\emptyset \cap B = \emptyset$ (Complement law)

e) $A \cup (B - A) = A \cup B$

$A \cup (B \cap \bar{A})$

$(A \cup B) \cap (A \cup \bar{A})$ distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws

$(A \cup B) \cap U$

$A \cup B$

17. Show that if A and B are sets in a universe U then $A \subseteq B$ if and only if $\overline{A} \cup B = U$.

$A \subseteq B \rightarrow \overline{A} \cup B = U$

let $A \subseteq B$

So $x \in A \rightarrow x \in B$

$\because$ every set is a subset of the universe

$\overline{A} \cup B = U$

Now we want to approve that

$U \subseteq \overline{A} \cup B$

let $x \in U$

$x \in A$ or $x \notin A$

- Case 1: $x \in A$

  $x \in B$

  $x \in \overline{A} \cup B$

- Case 2: $x \notin A$

  $x \in \overline{A}$

  $x \in \overline{A} \cup B$

  $\therefore U \subseteq \overline{A} \cup B$

$\overline{A} \cup B = U \rightarrow A \subseteq B$

let $\overline{A} \cup B = U$

So $x \in A$

$x \in U$

$x \in \overline{A} \cup B$

$x \in B$

$A \subseteq B$

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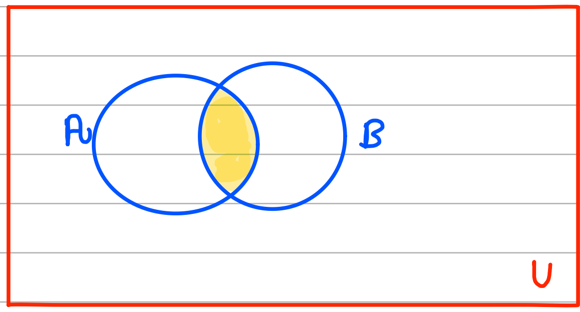
18. Given sets A and B in a universe U, draw the Venn diagrams of each of these sets.

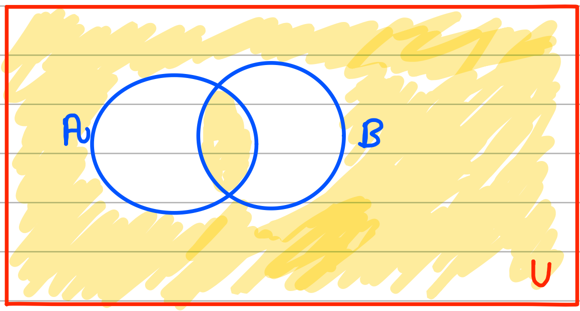
a) $A \rightarrow B = \{x \in U \mid x \in A \rightarrow x \in B\}$

b) $A \leftrightarrow B = \{x \in U \mid x \in A \leftrightarrow x \in B\}$

a)



b) $A \longleftrightarrow B$



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20. Let A, B, and C be sets. Show that

a) $(A \cup B) \subseteq (A \cup B \cup C)$.
b) $(A \cap B \cap C) \subseteq (A \cap B)$.
c) $(A - B) - C \subseteq A - C$.
d) $(A - C) \cap (C - B) = \emptyset$.
e) $(B - A) \cup (C - A) = (B \cup C) - A$.

a) $(A \cup B) \subseteq (A \cup B \cup C)$

let $x \in (A \cup B)$

$x \in A$ or $x \in B$

$x \in (A \cup B \cup C)$

b) $(A \cap B \cap C) \subseteq (A \cap B)$

let $x \in (A \cap B \cap C)$

$x \in A$ and $x \in B$ and $x \in C$

$x \in (A \cap B)$

c) $(A - B) - C \subseteq A - C$

let $x \in (A - B) - C$

$x \in (A - B)$ and $x \notin C$

$x \in A$ and $x \notin B$ and $x \notin C$

$x \in A$ and $x \notin C$

$x \in A - C$

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d) $(A - C) \cap (C - B) = \emptyset$

$(A \cap \overline{C}) \cap (C \cap \overline{B})$

$= A \cap \overline{B} \cap (C \cap \overline{C})$

$= (A \cap \overline{B}) \cap \emptyset$

$= \emptyset$

e) $(B - A) \cup (C - A) = (B \cup C) - A$

$= (B \cap \overline{A}) \cup (C \cap \overline{A})$

$= (B \cup C) \cap \overline{A}$

$= (B \cup C) - \overline{\overline{A}}$

$= (B \cup C) - A$

21. Show that if A and B are sets, then

a) $A - B = A \cap \overline{B}$.

b) $(A \cap B) \cup (A \cap \overline{B}) = A$.

a) $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

$\therefore x \in A \text{ and } x \in \overline{B}$

$x \in A \cap \overline{B}$

$\therefore A - B = A \cap \overline{B}$

$A \cap \overline{B} = \{x \mid x \in A \text{ and } x \in \overline{B}\}$

$\therefore x \in A \text{ and } x \notin B$

$x \in A - B$

$\therefore A - B = A \cap \overline{B}$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

$(A \cap B) \cup (A \cap \overline{B})$

$= A \cap (B \cup \overline{B})$

$= A \cap U$

$= A$

$\therefore (A \cap B) \cup (A \cap \overline{B}) = A$

22. Show that if A and B are sets with $A \subseteq B$, then

a) $A \cup B = B$.

b) $A \cap B = A$.

a) let $x \in (A \cup B)$

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$x \in B \text{ or } x \in B$

$x \in B$

$B \subseteq (A \cup B)$

let $x \in B$

$x \in (A \cup B)$

$\therefore A \cup B = B$

b) $A \cap B = A$

$A \cap B \subseteq A$

$x \in (A \cap B)$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$x \in A$

$A \subseteq (A \cap B)$

$x \in A$

$x \in B$

$x \in (A \cap B)$

$\therefore A \cap B = A$

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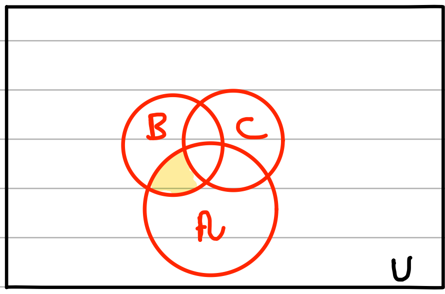
29. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

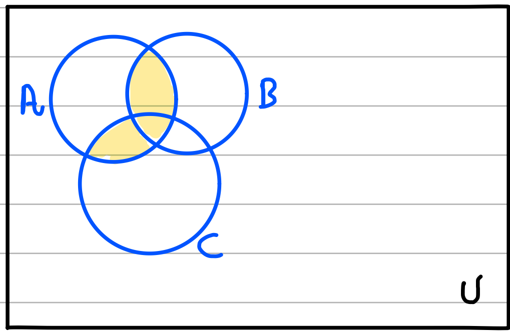
a) $A \cap (B - C)$ **b)** $(A \cap B) \cup (A \cap C)$

c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

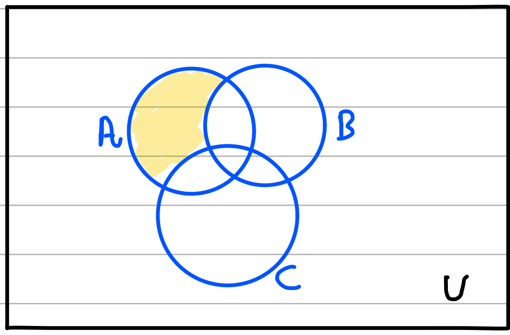
a)



b)



c)



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38. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

$\{1,3,5\} \oplus \{1,2,3\} =$

$\{1,3,5\} \cup \{1,2,3\} = \{1,2,3,5\}$

$\{1,3,5\} \cap \{1,2,3\} = \{1,3\}$

$\{1,3,5\} \oplus \{1,2,3\} = \{2,5\}$

43. Show that if A is a subset of a universal set U, then

a) $A \oplus A = \emptyset$.

b) $A \oplus \emptyset = A$.

c) $A \oplus U = \overline{A}$.

d) $A \oplus \overline{A} = U$.

a) $(A - A) \cup (A - A) = \emptyset \cup \emptyset = \emptyset$

b) $(A - \emptyset) \cup (\emptyset - A) = A \cup \emptyset = A$

c) $(A - U) \cup (U - A) = \emptyset \cup \overline{A} = \overline{A}$

d) $(A - \overline{A}) \cup (\overline{A} - A) = A \cup \overline{A} = U$

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