In Problems 1 and 2, $y = 1/(1 + c_1 e^{-x})$ is a one-parameter family of solutions of the first-order DE $y' = y - y^2$. Find a solution of the first-order IVP consisting of this differential equation and the given initial condition.

2. $y(-1) = 2$

$$2 = \frac{1}{1 + C_1 e^{-(-1)}}$$

$$2 = \frac{1}{1 + C_1 e^1}$$

$$2(1 + C_1 e^1) = 1$$

$$2 + 2C_1 e = 1$$

$$2C_1 e = 1 - 2$$

$$C_1 = -\frac{1}{2e}$$

$$y = \frac{1}{1 - \frac{1}{2e} e^{-x}}$$

$$= \frac{1}{1 - \frac{e^{-1} e^{-x}}{2}}$$

$$= \frac{1}{1 - \frac{e^{-1-x}}{2}}$$

In Problems 3–6, $y = 1/(x^2 + c)$ is a one-parameter family of solutions of the first-order DE $y' + 2xy^2 = 0$. Find a solution of the first-order IVP consisting of this differential equation and the given initial condition. Give the largest interval I over which the solution is defined.

4. $y(-2) = \frac{1}{2}$

$$Y = \frac{1}{X^2 + C}$$

$$\frac{1}{2} = \frac{1}{-2^2 + C}$$

$$\frac{1}{2} = \frac{1}{4 + C}$$

$4 + C = 2$

$C = 2 - 4 = -2$

$C = -2$

$$Y = \frac{1}{X^2 - 2}$$

$X^2 - 2 = 0$

$X^2 = 2$

$\sqrt{X^2} = \sqrt{2}$

$X = \pm\sqrt{2}$

The larges interval: $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, +\infty)$

In Problems 7–10, $x = c_1 \cos t + c_2 \sin t$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$. Find a solution of the second-order IVP consisting of this differential equation and the given initial conditions.

8. $x(\pi/2) = 0, \quad x'(\pi/2) = 1$

$0 = C_1 \cos \frac{\pi}{2} + C_2 \sin \frac{\pi}{2}$

$0 = C_1(0) + C_2(1)$

$C_2 = 0$

$x' = -C_1 \sin t + C_2 \cos t$

$1 = -C_1 \sin \frac{\pi}{2} + C_2 \cos \frac{\pi}{2}$

$1 = -C_1(1) + 0$

$C_1 = -1$

$x = -\cos t + (0) \sin t$

$x = -\cos t$

In Problems 11–14, $y = c_1e^x + c_2e^{-x}$ is a two-parameter family of solutions of the second-order DE $y'' - y = 0$. Find a solution of the second-order IVP consisting of this differential equation and the given initial conditions.

12. $y(1) = 0, \quad y'(1) = e$

$0 = C_1e^1 + C_2e^{-1}$

$0 = C_1e^1 + C_2e^{-1} \rightarrow (1)$

$y' = C_1e^x - C_2e^{-x}$

$e = C_1e^1 - C_2e^{-1} \rightarrow (2)$

$0 = C_1e^1 + C_2e^{-1}$

$e = C_1e^1 - C_2e^{-1}$

$e = 2C_1e$

$C_1 = \frac{1}{2}$

$0 = C_1e^1 + C_2e^{-1}$

$0 = \frac{1}{2}e^1 + \frac{C_2}{e}$

$\frac{C_2}{e} = -\frac{e}{2}$

$C_2 = -\frac{e^2}{2}$

$$y = \frac{1}{2} e^{x} - \frac{e^{2}}{2} e^{-x}$$

$$y = \frac{1}{2} e^{x} - \frac{e^{2-x}}{2}$$

In Problems 15 and 16 determine by inspection at least two solutions of the given first-order IVP.

16. $xy' = 2y, \quad y(0) = 0$

$Y = 0$ and $Y = X^2$

at $Y = 0$

$y' = 0$

$X(0) = 2(0)$

$0 = 0$

$Y(0) = 0$

at $Y = X^2$

$y' = 2X$

$X(2X) = 2(X^2)$

$2X^2 = 2X^2$

$0 = 0$

$Y(0) = 0$

$\therefore Y = 0$ and $Y = X^2$ The Solution