Exercise set (4.5):

8-13(d)-14(d)-
16(b,c)-17-23-24-
28-33(a)

5–8 Use the given values of a and b to express the following limits as integrals. (Do not evaluate the integrals.)

8. $\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (\sin^2 x_k^*)\Delta x_k;\ a = 0, b = \pi/2$

$$\int_0^{\pi/2} \sin^2 x \, dx$$

13–16 Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed.

13. (d) $\int_{-5}^{5} x\,dx$

$Y = X$

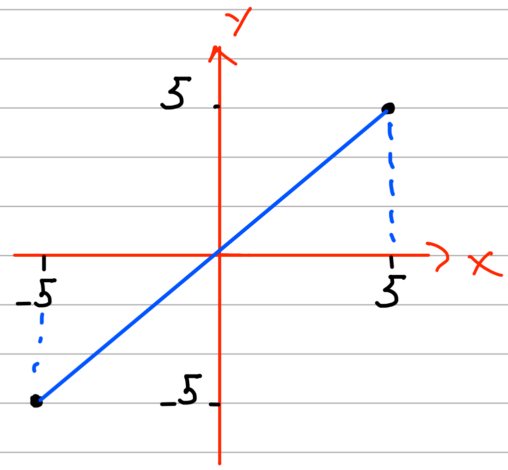
at $X = -5 \longrightarrow Y = -5$ $(-5, -5)$

at $X = 5 \longrightarrow Y = 5$ $(5, 5)$

$A =$ area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$



$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(5)(5) - \frac{1}{2}(5)(5)$

$= 0$

14. (a) $\int_0^2 \left(1 - \frac{1}{2}x\right)\, dx$ (b) $\int_{-1}^1 \left(1 - \frac{1}{2}x\right)\, dx$

(c) $\int_2^3 \left(1 - \frac{1}{2}x\right)\, dx$ (d) $\int_0^3 \left(1 - \frac{1}{2}x\right)\, dx$

d) $\int_0^3 (1-\frac{1}{2}x)dx$

$Y = 1 - \frac{1}{2}X$

at $X=0 \rightarrow Y = 1 - \frac{1}{2}(0) = 1$ $(0, 1)$

at $X=3 \rightarrow Y = 1 - \frac{1}{2}(3) = -\frac{1}{2}$ $(3, -\frac{1}{2})$

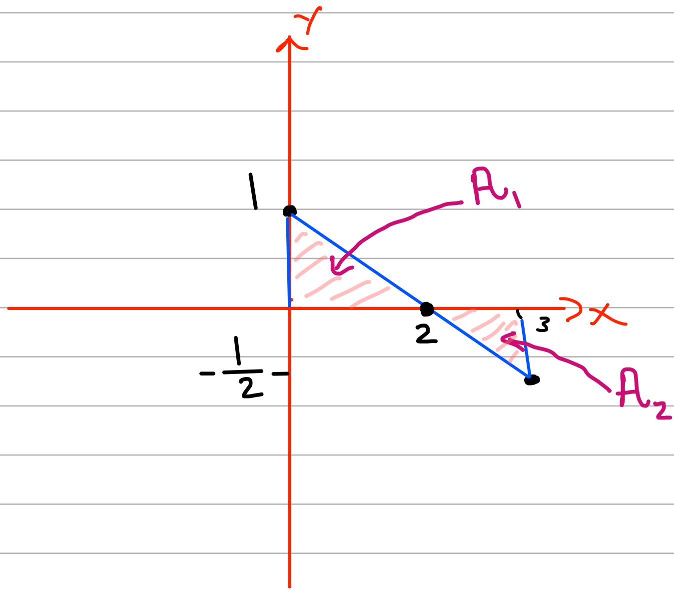
$Y=0 \rightarrow 0 = 1 - \frac{1}{2}X \rightarrow \frac{X}{2} = 1$

$X = 2$

A = area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$



$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(1) - \frac{1}{2}(1)(\frac{1}{2})$

$= 1 - \frac{1}{4} = \frac{4.1 - 1.1}{4} = \frac{3}{4}$

16. (b) $\int_{-\pi/3}^{\pi/3} \sin x \, dx$

$Y = \sin X$

at $X = \pi/3 \longrightarrow Y = \sin(\pi/3) = \sqrt{3}/2$ $(\pi/3, \sqrt{3}/2)$

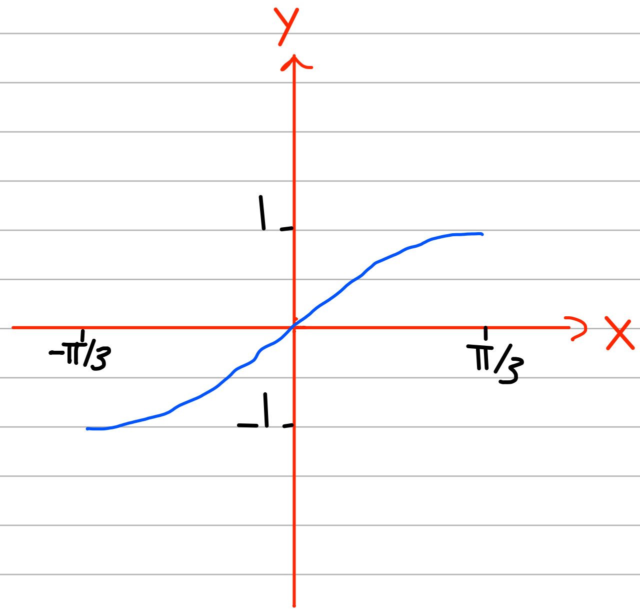
at $X = -\pi/3 \longrightarrow Y = \sin(-\pi/3) = -\sqrt{3}/2$ $(-\pi/3, -\sqrt{3}/2)$

A = area of Circle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{4}\pi(r_1^2) - \frac{1}{4}\pi(r_2^2)$



$= \frac{1}{4}\pi(1^2) - \frac{1}{4}\pi(-1^2)$

$= \frac{1}{4}\pi - \frac{1}{4}\pi$

$= 0$

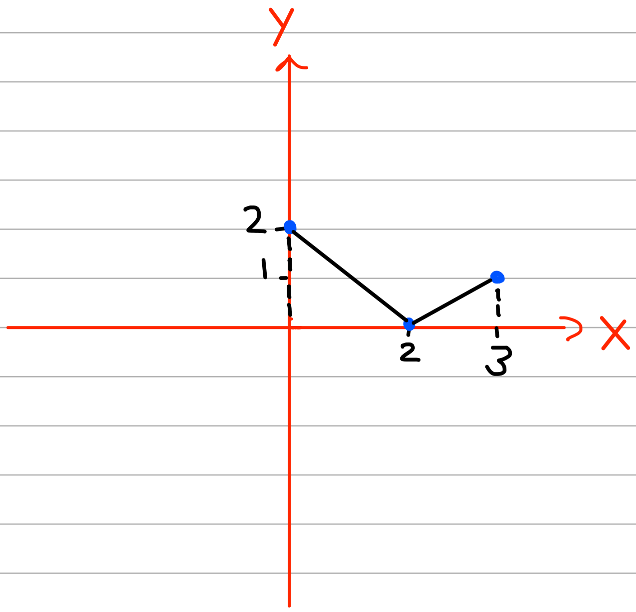
$A_1 = A_2$

(c) $\int_0^3 |x-2|\,dx$

$Y = |x - 2|$

at $X = 0 \longrightarrow Y = |0 - 2| = 2$ (0 , 2)

at $X = 3 \longrightarrow Y = |3 - 2| = 1$ (3 , 1)



$A =$ area of triangle

$= A_1 + A_2$

$= \frac{1}{2}(b_1)(h_1) + \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$= 2 + \frac{1}{2}$

$= \frac{5}{2}$

17. In each part, evaluate the integral, given that

$$f(x) = \begin{cases} |x-2|, & x \geq 0 \\ x+2, & x < 0 \end{cases}$$

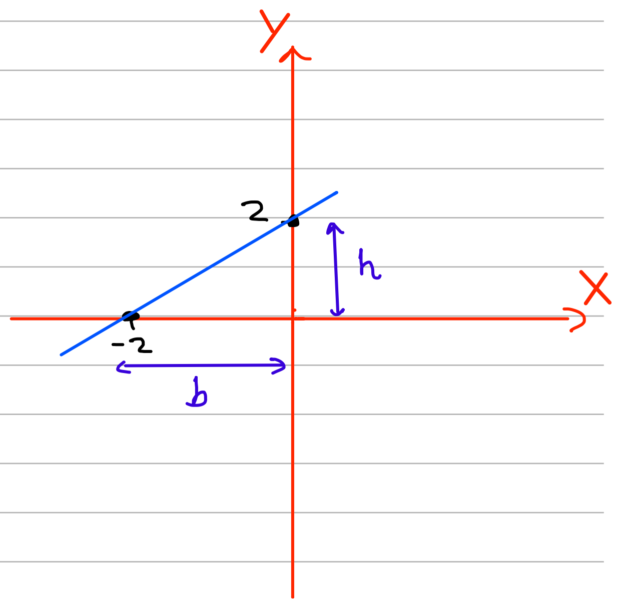
(a) $\int_{-2}^{0} f(x)\,dx$

$Y = X + 2$

at $X = -2 \longrightarrow Y = -2 + 2 = 0$ $(-2, 0)$

at $X = 0 \longrightarrow Y = 0 + 2 = 2$ $(0, 2)$

A = area of triangle



$= \frac{1}{2}(\text{base} \cdot \text{height})$

$= \frac{1}{2}(2)(2)$

$= \frac{4}{2}$

$= 2$

(b) $\int_{-2}^{2} f(x)\,dx$ ✱

$\int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx$

$= \int_{-2}^{0} x+2\, dx + \int_{0}^{2} |x-2| dx$

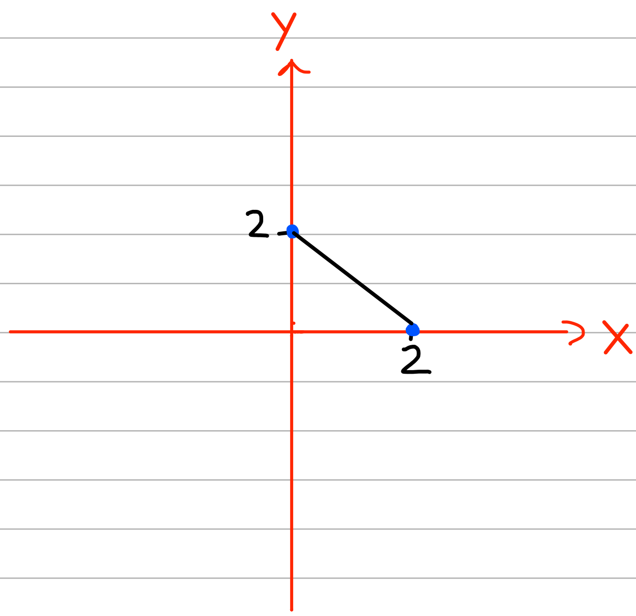
$\int_{-2}^{0} x+2\, dx = 2$

$\int_{0}^{2} |x-2| dx$

$y = |x-2|$

at $x = 0 \longrightarrow y = |0-2| = 2$ $(0, 2)$

at $x = 2 \longrightarrow y = |2-2| = 0$ $(2, 0)$



A = area of triangle

$= \frac{1}{2}(b)(h) = \frac{1}{2}(2)(2)$

$= 2$

$\int_{-2}^{0} x+2\, dx + \int_{0}^{2} |x-2| dx$

$= 2 + 2 = 4$

(c) $\int_0^6 f(x)\,dx$

$= \int_0^6 |x-2|\,dx$

$y = |x-2|$

$y = 0 \rightarrow 0 = x-2 \rightarrow \boxed{x=2}$

at $x = 0 \rightarrow y = |0-2| = |-2| = 2$ $(0, 2)$

at $x = 6 \rightarrow y = |6-2| = |4| = 4$ $(6, 4)$

A = area of triangle

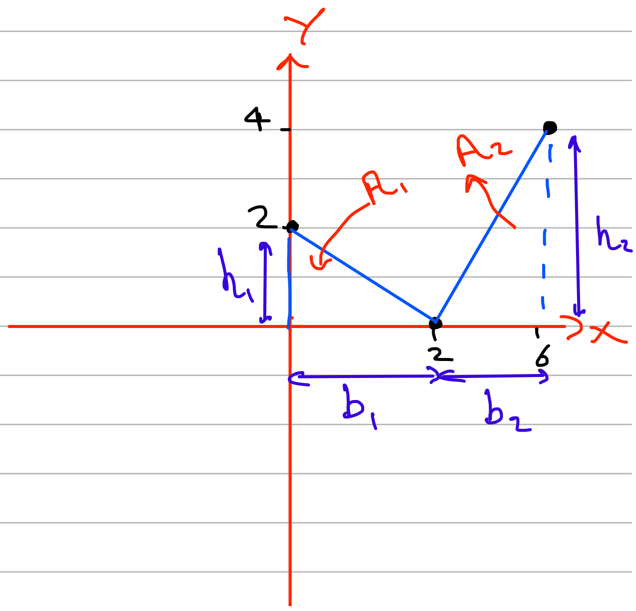
$= A_1 + A_2$

$= \frac{1}{2}(b_1)(h_1) + \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(2) + \frac{1}{2}(4)(4)$

$= 2 + 8$

$= 10$



(d) $\int_{-4}^{6} f(x)\,dx$

$$\int_{-4}^{6} f(X)\,dX = \int_{-4}^{0} f(X)\,dX + \int_{0}^{6} f(X)\,dX$$

$$= \int_{-4}^{0} (X+2)\,dX + \int_{0}^{6} |X-2|\,dX$$

$$= \int_{-4}^{0} (X+2)\,dX + 10$$

$Y = X+2$

at $X = 0 \rightarrow Y = 0+2 = 2$ $(0, 2)$

at $X = -4 \rightarrow Y = -4+2 = -2$ $(-4, -2)$

A = area of triangle

$= A_1 + (-A_2)$

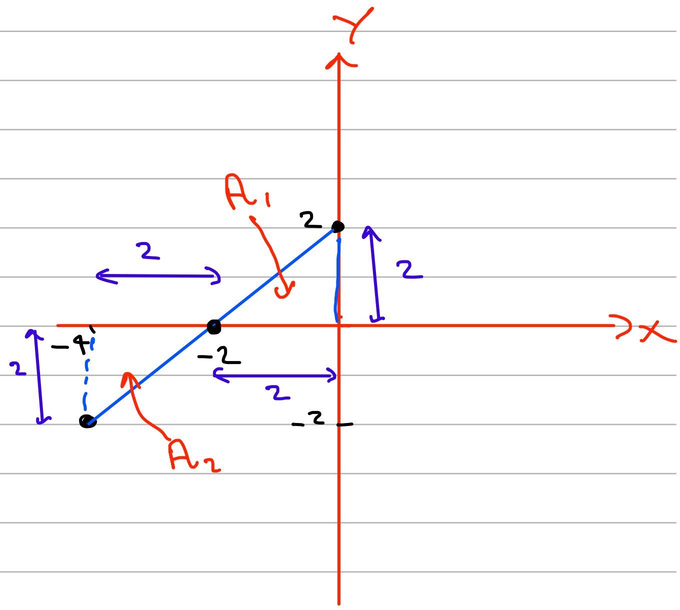
$= A_1 - A_2$

$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(2) - \frac{1}{2}(2)(2)$

$= 2 - 2$

$= 0$



$$\int_{-4}^{6} f(X)\,dX = \int_{-4}^{0} (X+2)\,dX + 10$$

$$= 0 + 10 = 10$$

23. Find $\int_1^5 f(x)\,dx$ if

$$\int_0^1 f(x)\,dx = -2 \quad \text{and} \quad \int_0^5 f(x)\,dx = 1$$

$$\int_0^5 f(x)dx = \int_0^1 f(x)dx + \int_1^5 f(x)dx$$

$$1 = -2 + \int_1^5 f(x)dx$$

$$1 + 2 = \int_1^5 f(x)dx$$

$$\int_1^5 f(x)dx = 3$$

24. Find $\int_{3}^{-2} f(x)\,dx$ if

$$\int_{-2}^{1} f(x)\,dx = 2 \quad \text{and} \quad \int_{1}^{3} f(x)\,dx = -6$$

$$\int_{3}^{-2} f(x)dx = -\int_{-2}^{3} f(x)dx$$

$$= -\left[\int_{-2}^{1} f(x)dx + \int_{1}^{3} f(x)dx\right]$$

$$= -(2+(-6))$$

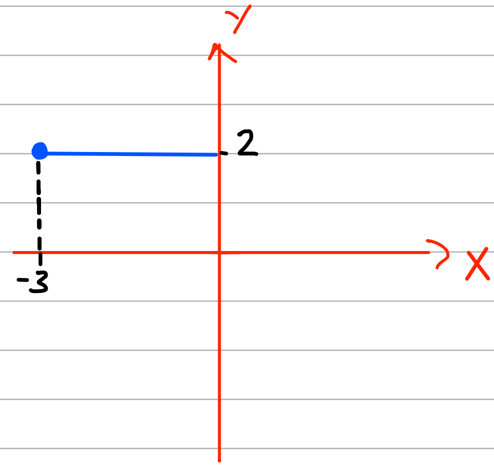
$$= -(2-6)$$

$$= 4$$

25–28 Use Theorem 4.5.4 and appropriate formulas from geometry to evaluate the integrals.

28. $\int_{-3}^{0} (2 + \sqrt{9 - x^2})\, dx$

$\int_{-3}^{0} 2\,dx + \int_{-3}^{0} \sqrt{9 - x^2}\,dx$



$\int_{-3}^{0} 2\,dx = 2 \cdot 3 = 6$

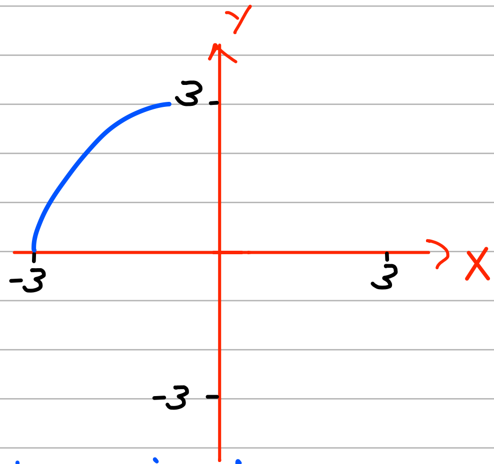
$\int_{-3}^{0} \sqrt{9 - x^2}\,dx =$

$y = \sqrt{9 - x^2}$

$y^2 = (\sqrt{9 - x^2})^2$

$y^2 = 9 - x^2$

$y^2 + x^2 = 9$



$\int_{-3}^{0} \sqrt{9 - x^2}\,dx =$ area of quarter circle

$= \frac{1}{4} \pi r^2$

$= \frac{1}{4} \pi (3)^2 = \frac{9\pi}{4}$

$\int_{-3}^{0} 2\,dx + \int_{-3}^{0} \sqrt{9 - x^2}\,dx$

$= 6 + \frac{9\pi}{4}$

33–34 Use Theorem 4.5.6 to determine whether the value of the integral is positive or negative.

33. (a) $\int_2^3 \frac{\sqrt{x}}{1-x}\,dx$

$\sqrt{X} \geqslant 0 \longrightarrow X \geqslant 0$

$1 - X \longrightarrow X \geqslant 2 \longrightarrow (1-X) < 0$

$f(X) = \frac{\sqrt{X}}{1-X} < 0$ on $[2,3]$

So the integral is negative on $[2,3]$

(b) $\int_0^4 \frac{x^2}{3 - \cos x}\, dx$

$x^2 \geqslant 0$ for all x

$-1 \leqslant \cos x \leqslant 1$

$1 \geqslant -\cos x \geqslant -1$

$3+1 \geqslant 3-\cos x \geqslant -1+3$

$4 \geqslant 3-\cos x \geqslant 2$

$\therefore 3-\cos x > 0$ for all x

$f(x) \geqslant 0$ on $[0,4]$

$$\int_0^4 \frac{x^2}{3-\cos x}\, dx \geqslant 0$$

So the integral is Positive

34. (a) $\int_{-3}^{-1} \frac{x^4}{\sqrt{3-x}}\,dx$

$x^4 \geqslant 0 \longrightarrow x \geqslant 0$

$\sqrt{3-x} \geqslant 0$ on $[-1,-3]$

So the integral is Positive on $[-1,-3]$

(b) $\int_{-2}^{2} \frac{x^3-9}{|x|+1}\,dx$

$x^3-9 < 0$ on $[-2,2]$

$|x|+1 > 0$ on $[-2,2]$

So the integral is negative on $[-2,2]$