IQ
ACADEMY

In Problems 1–22 solve the given differential equation by separation of variables.

**2.** $\dfrac{dy}{dx} = (x + 1)^2$

$dy = (x+1)^2 dx$

$\int dy = \int (x+1)^2 dx$

$y = \dfrac{(x+1)^3}{3} + C$

**4.** $dy - (y-1)^2 dx = 0$

$$dy = (y-1)^2 dx$$

$$\frac{dy}{(y-1)^2} = dx$$

$$\int dx = \int \frac{dy}{(y-1)^2}$$

$$\int \frac{1}{x^n} dx = -\frac{1}{(n-1)\, x^{n-1}}$$

$$x + C = -\frac{1}{(2-1)(y-1)}$$

$$x + C = -\frac{1}{(y-1)}$$

$$(y-1)(x+c) = -\frac{1}{(y-1)}(y-1)$$

$$yx + yc - x - c = -1$$

$$yx + yc = -1 + x + c$$

$$y\,(x+c) = -1 + x + c$$

$$y = \frac{-1 + x + c}{x+c}$$

$$y = -\frac{1}{x+c} + \frac{x+c}{x+c}$$

$$y = -\frac{1}{x+c} + 1$$

**6.** $\frac{dy}{dx} + 2xy^2 = 0$

$\frac{dy}{dx} = -2xy^2$

$dy = -2xy^2 \, dx$

$\frac{1}{y^2} dy = -2x \, dx$

$\int \frac{1}{y^2} dy = \int -2x \, dx$

$-\frac{1}{y} = -2 \frac{x^2}{2} + C$

$-\frac{1}{y} = -x^2 + C$

$\frac{1}{y} = -(-x^2 + C)$

$\frac{1}{y} = x^2 - C$

$y = \frac{1}{x^2 - C}$

**8.** $e^x y \dfrac{dy}{dx} = e^{-y} + e^{-2x-y}$

$$e^x y \frac{dy\cancel{dx}}{\cancel{dx}} = (e^{-y} + e^{-2x-y})\,dx$$

$$e^x y\,dy = e^{-y} + e^{-2x} e^{-y} \quad dx$$

$$e^x y\,dy = e^{-y}(1 + e^{-2x})\,dx$$

$$\frac{e^x y\,dy}{e^{-y}} = \frac{\cancel{e^{-y}}(1 + e^{-2x})}{\cancel{e^{-y}}}\,dx$$

$$\frac{e^x y\,dy}{e^{-y}} = (1 + e^{-2x})\,dx$$

$$\frac{\cancel{e^x} y\,dy}{\cancel{e^x} e^{-y}} = \frac{(1 + e^{-2x})}{e^x}\,dx$$

$$y e^y\,dy = (e^{-x} + e^{-3x})\,dx$$

$$\int y e^y\,dy = \int (e^{-x} + e^{-3x})\,dx$$

$u = y \qquad dv = e^y\,dy$

$du = dy \qquad v = e^y$

$$y e^y - e^y = -e^{-x} - \frac{1}{3} e^{-3x} + C$$

$$\int y e^y\,dy = y e^y - \int e^y\,dy$$
$$= y e^y - e^y$$

**14.** $x(1+y^2)^{1/2}\,dx = y(1+x^2)^{1/2}\,dy$

$$\frac{x\cancel{(1+y^2)^{1/2}}}{\cancel{(1+y^2)^{1/2}}\,(1+x^2)^{1/2}}\,dx = \frac{y\cancel{(1+x^2)^{1/2}}}{(1+y^2)^{1/2}\,\cancel{(1+x^2)^{1/2}}}\,dy$$

$$\frac{x}{(1+x^2)^{1/2}}\,dx = \frac{y}{(1+y^2)^{1/2}}\,dy$$

$$\int \frac{x}{(1+x^2)^{1/2}}\,dx = \int \frac{y}{(1+y^2)^{1/2}}\,dy$$

$u = 1 + x^2$

$du = 2x\,dx \longrightarrow \frac{du}{2} = x\,dx$

$$\frac{1}{2}\int \frac{du}{u^{1/2}} = \frac{1}{2} \cdot - \frac{1}{(\frac{1}{2}-1)\,u^{\frac{1}{2}-1}} = \frac{1}{2} \cdot - \frac{1}{-\frac{1}{2}u^{-1/2}}$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{1}{2}u^{-1/2}} = \frac{1}{2} \cdot \frac{u^{1/2}}{\frac{1}{2}}$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2}u^{1/2} = u^{1/2} = (1+x^2)^{1/2}$$

$$\int \frac{x}{(1+x^2)^{1/2}}\,dx = \int \frac{y}{(1+y^2)^{1/2}}\,dy$$

$$(1+x^2)^{1/2} + C = (1+y^2)^{1/2}$$

**16.** $\frac{dQ}{dt} = k(Q - 70)$

$dQ = K(Q - 70)\,dt$

$\frac{dQ}{Q - 70} = K\,dt$

$\int \frac{dQ}{Q - 70} = \int K\,dt$

$\int \frac{dQ}{Q - 70} =$

$u = Q - 70$ , $du = dQ$

$\int \frac{1}{u}\,du = \ln|u| = \ln|Q - 70|$

$\int \frac{dQ}{Q - 70} = \int K\,dt$

$\ln|Q - 70| = Kt + C$

In Problems 23–28 find an explicit solution of the given initial-value problem.

**26.** $\frac{dy}{dt} + 2y = 1, \quad y(0) = \frac{5}{2}$

$\frac{dY}{dt} = 1 - 2Y$

$dY = (1 - 2Y)\,dt$

$\frac{dY}{1 - 2Y} = dt$

$\int \frac{1}{1 - 2Y}\,dY = \int dt$

$u = 1 - 2Y$

$du = -2\,dY$

$\frac{du}{-2} = dY$

$-\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| = -\frac{1}{2}\ln|1 - 2Y|$

$\int \frac{1}{1 - 2Y}\,dY = \int dt$

$-\frac{1}{2}\ln|1 - 2Y| = t + C$

$1 - 2Y = C_1 e^{-2t}$

$1 - \cancel{2}\,\frac{5}{\cancel{2}} = C_1 e^{-2(0)}$

$1 - 5 = C_1 \longrightarrow C_1 = -4$

The solution of initial Value Problem

$1-2Y=-4e^{-2t}$

$Y=2e^{-2t}+\frac{1}{2}$