IQ ACADEMY

# Exercise set (1.1)

In Problems 1–8 state the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear by matching it with (6).

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x). \qquad (6)$$

**2.** $x\dfrac{d^3y}{dx^3} - \left(\dfrac{dy}{dx}\right)^4 + y = 0$

Third order ordinary differential equation

its nonlinear because its from the fourth degree

**4.** $\dfrac{d^2u}{dr^2} + \dfrac{du}{dr} + u = \cos(r + u)$

second order ordinary differential equation

its nonlinear because Cos not linear function

**6.** $\dfrac{d^2R}{dt^2} = -\dfrac{k}{R^2}$

second order ordinary differential equation

its nonlinear because its from the fourth degree

**8.** $\ddot{x} - \left(1 - \frac{\dot{x}^2}{3}\right)\dot{x} + x = 0$

second order ordinary differential equation

its nonlinear because its from the second degree

In Problems 9 and 10 determine whether the given first-order differential equation is linear in the indicated dependent variable by matching it with the first differential equation given in (7).

**10.** $u\,dv + (v + uv - ue^{u})\,du = 0$; in $v$; in $u$

in V

$$u\frac{dv}{du} + (V + uV - ue^{u})\frac{du}{du} = \frac{0}{du}$$

$$u\frac{dv}{du} + (V + uV - ue^{u}) = 0$$

$$u\frac{dv}{du} + v(1+u) - ue^{u}$$

$\therefore$ its linear in V

in u:

$$u\frac{dv}{dv} + (V + uV - ue^{u})\frac{du}{dV} = \frac{0}{dV}$$

$$u + (V + uV - ue^{u})\frac{du}{dV} = 0$$

$$u + V\frac{du}{dV} + uV\frac{du}{dV} - ue^{u}\frac{du}{dV} = 0$$

$\therefore$ its not linear in u

In Problems 11–14 verify that the indicated function is an explicit solution of the given differential equation. Assume an appropriate interval $I$ of definition for each solution.

**12.** $\frac{dy}{dt} + 20y = 24; \quad y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

$$\frac{dy}{dt} = 0 - \frac{6}{5} e^{-20t} \cdot -20 = \frac{120}{5} e^{-20t} = 24 e^{-20t}$$

$$24 e^{-20t} + 20\left(\frac{6}{5} - \frac{6}{5} e^{-20t}\right) = 24$$

$$24 e^{-20t} + \frac{120}{5} - \frac{120}{5} e^{-20t} = 24$$

$$\cancel{24 e^{-20t}} + 24 \cancel{- 24 e^{-20t}} = 24$$

$$24 = 24$$

$\therefore$ Y is solution of defferential equation

In Problems 15–18 verify that the indicated function $y = \phi(x)$ is an explicit solution of the given first-order differential equation. Proceed as in Example 2, by considering $\phi$ simply as a *function,* give its domain. Then by considering $\phi$ as a *solution* of the differential equation, give at least one interval $I$ of definition.

**16.** $y' = 25 + y^2; \quad y = 5 \tan 5x$

In Problems 21–24 verify that the indicated family of functions is a solution of the given differential equation. Assume an appropriate interval $I$ of definition for each solution.

**24.** $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 12x^2;$

$y = c_1x^{-1} + c_2x + c_3x \ln x + 4x^2$

$$\frac{dY}{dX} = -C_1X^{-2} + C_2 + C_3\ln X + C_3X\frac{1}{X} + 8X$$

$$\frac{dY}{dX} = -C_1X^{-2} + C_2 + C_3\ln X + C_3 + 8X$$

$$\frac{d^2Y}{dX^2} = 2C_1X^{-3} + \frac{C_3}{X} + 8$$

$$\frac{d^3Y}{dX^3} = -6C_1X^{-4} - \frac{C_3}{X^2} = -6C_1X^{-4} - C_3X^{-2}$$

$$X^3\frac{d^3Y}{dX^3} + 2X^2\frac{d^2Y}{dX^2} - X\frac{dY}{dX} + Y = 12X^2$$

$$X^3(-6C_1X^{-4} - C_3X^{-2}) + 2X^2\left(2C_1X^{-3} + \frac{C_3}{X} + 8\right)$$

$$-X(-C_1X^{-2} + C_2 + C_3\ln X + C_3 + 8X)$$

$$+(C_1X^{-1} + C_2X + C_3X\ln X + 4X^2) = 12X^2$$

$$-6C_1X^{-1} - C_3X + 4C_1X^{-1} + 2C_3X + 16X^2 + C_1X^{-1} - C_2X$$

$$-C_3X\ln X - C_3X - 8X^2 + C_1X^{-1} + C_2X + C_3X\ln X + 4X^2 = 12X^2$$

$$\cancel{-6C_1X^{-1}}\cancel{+4C_1X^{-1}}\cancel{+C_1X^{-1}}\cancel{+C_1X^{-1}}\cancel{-C_3X}\cancel{+2C_3X}\cancel{-C_3X}$$

$$+16X^2-8X^2+4X^2=12X^2$$

$$12X^2=12X^2$$

$\therefore$ Y is solution of defferential equation

In Problems 27–30 find values of $m$ so that the function $y = e^{mx}$ is a solution of the given differential equation.

**30.** $2y'' + 7y' - 4y = 0$

$y' = e^{mx} \cdot m = m e^{mx}$

$y'' = m^2 e^{mx}$

$2(m^2 e^{mx}) + 7(m e^{mx}) - 4e^{mx} = 0$

$\frac{2m^2 \cancel{e^{mx}}}{\cancel{e^{mx}}} + \frac{7m \cancel{e^{mx}}}{\cancel{e^{mx}}} - \frac{4\cancel{e^{mx}}}{\cancel{e^{mx}}} = 0$

$2m^2 + 7m - 4 = 0$

$2m^2 + 8m - m - 4 = 0$

$2m(m+4) - m - 4 = 0$

$2m(m+4) - (m+4) = 0$

$(m+4)(2m-1) = 0$

$m+4 = 0 \rightarrow m = -4$

$2m - 1 = 0 \rightarrow 2m = 1 \rightarrow m = \frac{1}{2}$

In Problems 31 and 32 find values of $m$ so that the function $y = x^m$ is a solution of the given differential equation.

**32.** $x^2y'' - 7xy' + 15y = 0$

$y' = mx^{m-1}$

$y'' = (m-1)mx^{m-2} = (m^2 - m)x^{m-2}$

$x^2y'' - 7xy' + 15y = 0$

$x^2(m^2 - m)x^{m-2} - 7x(mx^{m-1}) + 15x^m = 0$

$x^m m^2 - mx^m - 7mx^m + 15x^m = 0$

$\frac{\cancel{x^m} m^2}{\cancel{x^m}} - \frac{m\cancel{x^m}}{\cancel{x^m}} - \frac{7m\cancel{x^m}}{\cancel{x^m}} + \frac{15\cancel{x^m}}{\cancel{x^m}} = 0$

$m^2 - m - 7m + 15 = 0$

$m^2 - 8m + 15 = 0$

$m^2 - 3m - 5m + 15 = 0$

$m(m-3) - 5(m-3) = 0$

$(m-3)(m-5) = 0$

$m - 3 = 0 \longrightarrow m = 3$

$m - 5 = 0 \longrightarrow m = 5$