Exercise set (0.2)

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27, 28, 29 (a, b, c, f), 30 (b, c, g), 31, 32, 34, 35, 41,42, 43, 44 , 59 (a, b, e, f) p.25-26.

27–28 Find formulas for $f+g$, $f-g$, fg, and f/g, and state the domains of the functions. ■

27. $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$

$(f+g)(X) = 2\sqrt{X-1} + \sqrt{X-1} = 3\sqrt{X-1}$

Domain:

$X - 1 \geqslant 0$

$X \geqslant 1$

$D_{f+g} = [1, +\infty)$

$(f-g)(X) = 2\sqrt{X-1} - \sqrt{X-1} = \sqrt{X-1}$

Domain:

$X - 1 \geqslant 0$

$X \geqslant 1$

$D_{f-g} = [1, +\infty)$

$(fg)(X) = (2\sqrt{X-1})(\sqrt{X-1}) = 2(X-1) = 2X - 2$

Domain:

$D_{fg} = [1, +\infty)$

- $(f/g)(x) = \dfrac{2\cancel{\sqrt{x-1}}}{\cancel{\sqrt{x-1}}} = 2$

Domain:

$x - 1 > 0$

$x > 1$

$D_{f/g} = [1, +\infty) \cap (1, +\infty) = (1, +\infty)$

28. $f(x) = \frac{x}{1+x^2}$, $g(x) = \frac{1}{x}$

$(f+g)(X) = \frac{X}{1+X^2} + \frac{1}{X}$

$= \frac{X^2 + (1+X^2)}{X(1+X^2)} = \frac{2X^2+1}{X(1+X^2)}$

Domain:

$D_f = (-\infty, +\infty)$

$D_g = (-\infty, 0) \cup (0, +\infty)$

$D_{f+g} = (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty)$

$= (-\infty, 0) \cup (0, +\infty)$

$(f-g)(X) = \frac{X}{1+X^2} - \frac{1}{X}$

$= \frac{X^2 - (1+X^2)}{X(1+X^2)} = \frac{-1}{X(1+X^2)}$

Domain:

$D_f = (-\infty, +\infty)$

$D_g = (-\infty, 0) \cup (0, +\infty)$

$D_{f-g} = (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty)$

$= (-\infty, 0) \cup (0, +\infty)$

- $(fg)(X) = \dfrac{\cancel{X}}{1+X^2} \cdot \dfrac{1}{\cancel{X}} = \dfrac{1}{1+X^2}$

Domain:

$D_f = (-\infty, +\infty)$

$D_g = (-\infty, 0) \cup (0, +\infty)$

$$D_{f \cdot g} = (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty)$$
$$= (-\infty, 0) \cup (0, +\infty)$$

- $(f/g)(X) = \dfrac{X}{1+X^2} \Big/ \dfrac{1}{X}$

$$= X \frac{X}{1+X^2} = \frac{X^2}{1+X^2}$$

Domain:

$D_f = (-\infty, +\infty)$

$D_g = (-\infty, 0) \cup (0, +\infty)$

$$D_{f/g} = (-\infty, +\infty) \cap (-\infty, 0) \cup (0, +\infty)$$
$$= (-\infty, 0) \cup (0, +\infty)$$

29. Let $f(x)=\sqrt{x}$ and $g(x)=x^3+1$. Find

(a) $f(g(2))$

$f(g(x)) = f(x^3+1) = \sqrt{x^3+1}$

$f(g(2)) = \sqrt{2^3+1} = \sqrt{8+1} = \sqrt{9} = 3$

(b) $g(f(4))$

$g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^3 + 1$

$g(f(4)) = (\sqrt{4})^3 + 1 = 8 + 1 = 9$

(c) $f(f(16))$

$f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}}$

$f(f(16)) = \sqrt{\sqrt{16}} = 2$

(f) $g(3+h)$.

$g(3+h) = (3+h)^3 + 1$

30. Let $g(x) = \sqrt{x}$. Find

(b) $g(\sqrt{x} + 2)$

$g(\sqrt{x} + 2) = \sqrt{\sqrt{x} + 2}$

(c) $3g(5x)$

$3g(5x) = 3\sqrt{5x}$

(g) $g(1/\sqrt{x}\,)$

$g(1/\sqrt{x}) = \sqrt{\frac{1}{\sqrt{x}}} = \frac{1}{\sqrt{\sqrt{x}}} = \frac{1}{\sqrt[4]{x}}$

31–34 Find formulas for $f \circ g$ and $g \circ f$, and state the domains of the compositions.

31. $f(x) = x^2, \; g(x) = \sqrt{1 - x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{1-x})$

$= (\sqrt{1-x})^2$

$= 1 - x$

$D_{f \circ g} = D_g \cap D_R$

$D_g = 1 - x \geqslant 0 \rightarrow x \leqslant 1 \rightarrow (-\infty, 1]$

$D_R = (-\infty, +\infty)$

$D_{f \circ g} = (-\infty, 1] \cap (-\infty, +\infty) = (-\infty, 1]$

$(g \circ f)(x) = g(f(x)) = g(x^2)$

$= \sqrt{1 - x^2}$

$D_{g \circ f} = D_f \cap D_R$

$D_f = (-\infty, +\infty)$

$D_R = 1 - x^2 \geqslant 0 \rightarrow 1 \geqslant x^2 \rightarrow \sqrt{1} \geqslant \sqrt{x^2}$

$|x| \leq 1 = -1 \leqslant x \leqslant 1 = [-1, 1]$

$D_{g \circ f} = (-\infty, +\infty) \cap [-1, 1] = [-1, 1]$

32. $f(x) = \sqrt{x-3}, \; g(x) = \sqrt{x^2+3}$

$-\ (f\circ g)(x) = f(g(x)) = f(\sqrt{x^2+3}) = \sqrt{\sqrt{x^2+3} - 3}$

$-\ D_{f\circ g} = D_g \cap D_R$

$D_g = (-\infty, +\infty)$

$D_R = \sqrt{x^2+3} - 3 \geqslant 0 \longrightarrow \sqrt{x^2+3} \geqslant 3$

$(\sqrt{x^2+3})^2 \geqslant (3)^2 \longrightarrow x^2 + 3 \geqslant 9$

$x^2 \geqslant 6 \longrightarrow \sqrt{x^2} \geqslant \sqrt{6}$

$|x| \geqslant \sqrt{6} \longrightarrow x \geqslant \sqrt{6}$ or $x \leqslant -\sqrt{6}$

$D_R = (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$

$D_{f\circ g} = (-\infty, +\infty) \cap (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$

$= (-\infty, -\sqrt{6}] \cup [\sqrt{6}, +\infty)$

$-\ (g\circ f)(x) = g(f(x)) = g(\sqrt{x-3}) = \sqrt{\sqrt{(x-3)}^2 - 3}$

$= \sqrt{x-3+3} \quad = \sqrt{x}$

- $D_{gof} = D_f \cap D_R$

$D_f = X - 3 \geqslant 0 \longrightarrow X \geqslant 3 = [3, +\infty)$

$D_R = X \geqslant 0 = [0, +\infty)$

$D_{gof} = [3, +\infty) \cap [0, +\infty) = [3, +\infty)$

34. $f(x) = \dfrac{x}{1+x^2}$, $g(x) = \dfrac{1}{x}$

– $(f \circ g)(x) = f(g(x)) = f(1/x) = \dfrac{1/x}{1+(1/x)^2}$

$$= \frac{1/x}{1 + 1/x^2} = \frac{1/x}{\frac{x^2+1}{x^2}}$$

$$= \frac{x^2}{x^2+1} \cdot \frac{1}{x}$$

– $D_{f\circ g} = D_g \cap D_R$

$D_g = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

$D_R = \mathbb{R} = (-\infty, +\infty)$

$D_{f\circ g} = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

– $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x}{1+x^2}\right)$

$$= \frac{1}{\frac{x}{1+x^2}} = \frac{1+x^2}{x}$$

$D_{g\circ f} = D_f \cap D_R$

$D_f = \mathbb{R} = (-\infty, +\infty)$

$D_R = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

$D_{g\circ f} = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

35–40 Express f as a composition of two functions; that is, find g and h such that $f = g \circ h$. [*Note:* Each exercise has more than one solution.]

35. (a) $f(x) = \sqrt{x+2}$

$g(X) = \sqrt{X}$

$h(X) = X + 2$

$g \circ h = g(h(x)) = g(X+2) = \sqrt{X+2}$

(b) $f(x) = |x^2 - 3x + 5|$

$g(X) = |X|$

$h(X) = X^2 - 3X + 5$

$g \circ h = g(h(x)) = g(X^2 - 3X + 5) = |X^2 - 3X + 5|$

41–44 True–False Determine whether the statement is true or false. Explain your answer.

41. The domain of $f + g$ is the intersection of the domains of f and g. (T)

42. The domain of $f \circ g$ consists of all values of x in the domain of g for which $g(x) \neq 0$. (F)

domain of fog Consist of all X in the domain of g

for which g(X) in the domain of f

43. The graph of an even function is symmetric about the y-axis. (T)

44. The graph of $y = f(x + 2) + 3$ is obtained by translating the graph of $y = f(x)$ right 2 units and up 3 units. (F)

bY translating the graPh left 2 units and

up 3 units

59. In each part, classify the function as even, odd, or neither.

(a) $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x) \longrightarrow$ even function

(b) $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 \neq f(x) \longrightarrow$ not even

$f(-x) = -f(x) \longrightarrow$ odd function

(e) $f(x) = \dfrac{x^5 - x}{1 + x^2}$

$$f(-x) = \frac{(-x)^5 - (-x)}{1 + (-x)^2} = \frac{-x^5 + x}{1 + x^2}$$

$$= -\frac{x^5 - x}{1 + x^2}$$

$= -f(x) \longrightarrow$ odd function

(f) $f(x) = 2$

$f(-x) = 2 = f(x) \longrightarrow$ even function