Exercise set (4.5):

Exercise 4.5 P.239: Quick check ex.3

P. 240: 14-15(b)-16(a-b)-17(a-c-d)-22-23-24-33(b)

✓ QUICK CHECK EXERCISES 4.5 (See page 242 for answers.)

3. Use the accompanying figure to evaluate

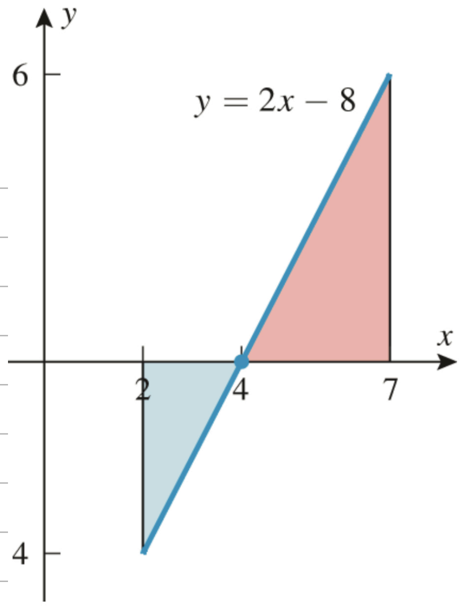
$$\int_2^7 (2x - 8)\, dx$$

$A = A_1 + (-A_2) = A_1 - A_2$

$= \frac{1}{2} b_1 h_1 - \frac{1}{2} b_2 h_2$

$= \frac{1}{2}(6)(3) - \frac{1}{2}(4)(2)$

$= 9 - 4 \quad = 5$



◀ **Figure Ex-3**

EXERCISE SET 4.5

13–16 Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed.

14. (a) $\int_0^2 \left(1 - \frac{1}{2}x\right)\, dx$ (b) $\int_{-1}^1 \left(1 - \frac{1}{2}x\right)\, dx$

(c) $\int_2^3 \left(1 - \frac{1}{2}x\right)\, dx$ (d) $\int_0^3 \left(1 - \frac{1}{2}x\right)\, dx$

$y = 1 - \frac{1}{2}x$

at $x = 0$

$y = 1 - \frac{1}{2}(0)$

$y = 1$ $(0, 1)$

at $x = 2$

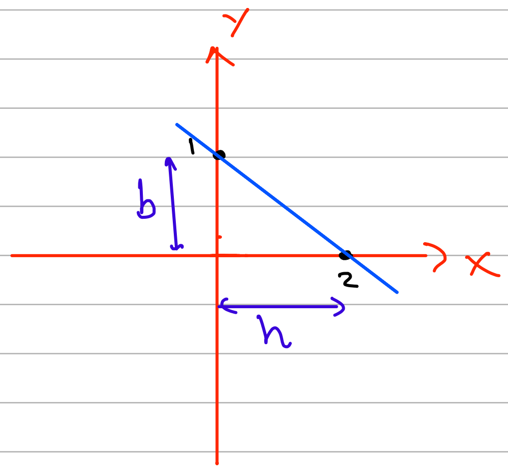
$y = 1 - \frac{1}{2}(2)$

$y = 0$ $(2, 0)$

A = area of triangle

$= \frac{1}{2}$ (base . height)

$= \frac{1}{2}(1)(2) = 1$



b) $\int_{-1}^{1} \left(1 - \frac{1}{2}x\right) dx$

$y = 1 - \frac{1}{2}x$

at $x = 1$

$y = 1 - \frac{1}{2}(1) = y = \left(\frac{1}{2}\right)$ $\quad \left(1, \frac{1}{2}\right)$

at $x = -1$

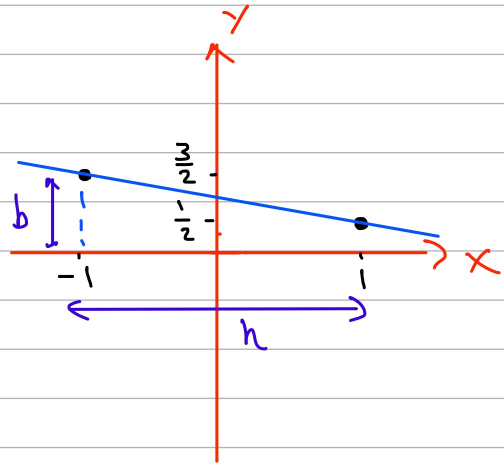
$y = 1 - \frac{1}{2}(-1) \Rightarrow y = \frac{3}{2}$ $\quad \left(-1, \frac{3}{2}\right)$

A = area of trapezoid

$= \frac{1}{2}(\text{base}1 + \text{base}2) \cdot \text{height}$

$= \frac{1}{2}\left(3/2 + \frac{1}{2}\right) \cdot 2$

$= 2$



c) $\int_2^3 \left(1 - \frac{1}{2}x\right) dx$

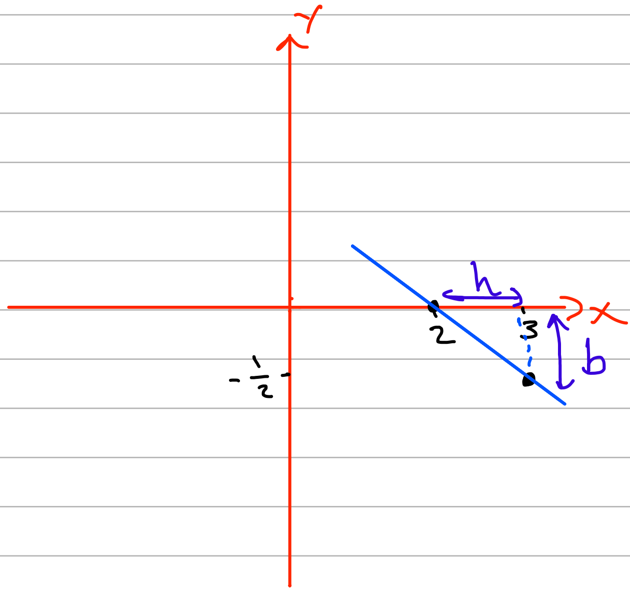
$y = 1 - \frac{1}{2}x$

at $x = 2$

$y = 1 - \frac{1}{2}(2) = 1 - 1 = 0$ $\quad (2, 0)$

at $x = 3$

$y = 1 - \frac{1}{2}(3) = -\frac{1}{2}$ $\quad (3, -\frac{1}{2})$



A = area of triangle

$= -\frac{1}{2}(\text{base} \cdot \text{height})$

$= -\frac{1}{2}\left(\frac{1}{2}\right)(1) = -\frac{1}{4}$

d) $\int_0^3 \left(1 - \frac{1}{2}x\right) dx$

$y = 1 - \frac{1}{2}x$

at $x = 0$

$y = 1 - \frac{1}{2}(0) \Longrightarrow y = 1 \qquad (0, 1)$

at $x = 3$

$y = 1 - \frac{1}{2}(3) \Longrightarrow y = -\frac{1}{2} \qquad (3, -\frac{1}{2})$

$y = 0 \Longrightarrow 0 = 1 - \frac{1}{2}x \Longrightarrow \frac{x}{2} = 1$

$x = 2$

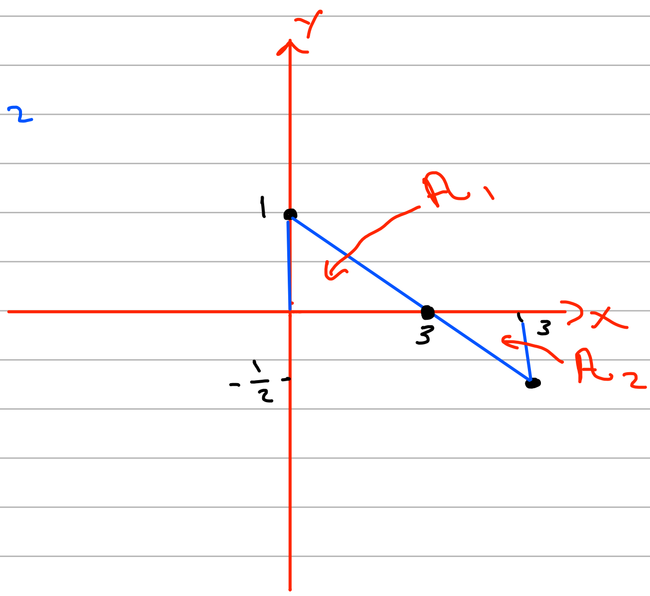
A = Area of triangle

$= A_1 + (-A_2) = A_1 - A_2$

$= \frac{1}{2}(b_1 h_1) - \frac{1}{2}(b_2 h_2)$

$= \frac{1}{2}(2 \cdot 1) - \frac{1}{2}(1 \cdot \frac{1}{2})$

$= 1 - \frac{1}{4} = \frac{3}{4}$



15. (b) $\int_0^{\pi} \cos x \, dx$

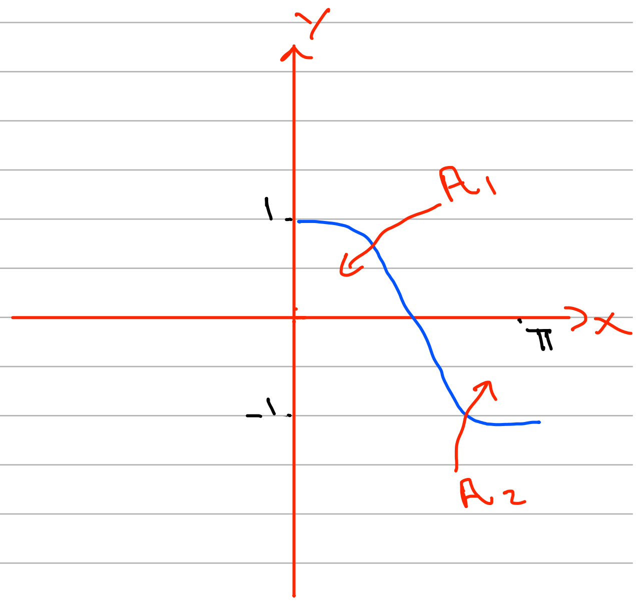
$Y = \cos x$

at $x = 0$

$Y = \cos 0 \Rightarrow Y = 1$ $(0, 1)$

at $x = \pi$

$Y = \cos \pi \Rightarrow Y = -1$ $(\pi, -1)$



A = area of Circle

$= A_1 - A_2 = \frac{1}{4}\pi r_1^2 - \frac{1}{4}\pi r_2^2$

$= \frac{1}{4}\pi(1)^2 - \frac{1}{4}\pi(-1)^2$

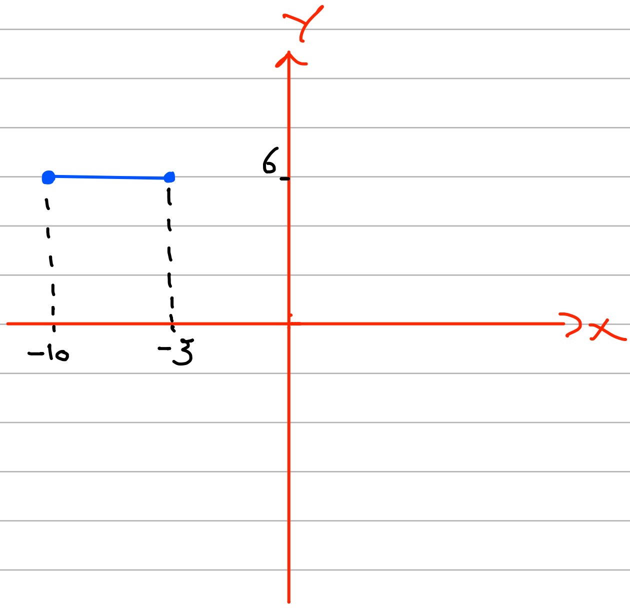
$= 0$

$A_1 = A_2$

16. (a) $\displaystyle\int_{-10}^{-5} 6\,dx$ (b) $\displaystyle\int_{-\pi/3}^{\pi/3} \sin x\,dx$

a) $Y = 6$

$A = 6 . 5 = 30$



b) $\int_{-\pi/3}^{\pi/3} \sin x \, dx$

$Y = \sin x$

at $x = \pi/3$

$Y = \sin(\pi/3) = \frac{\sqrt{3}}{2}$

at $x = -\pi/3$

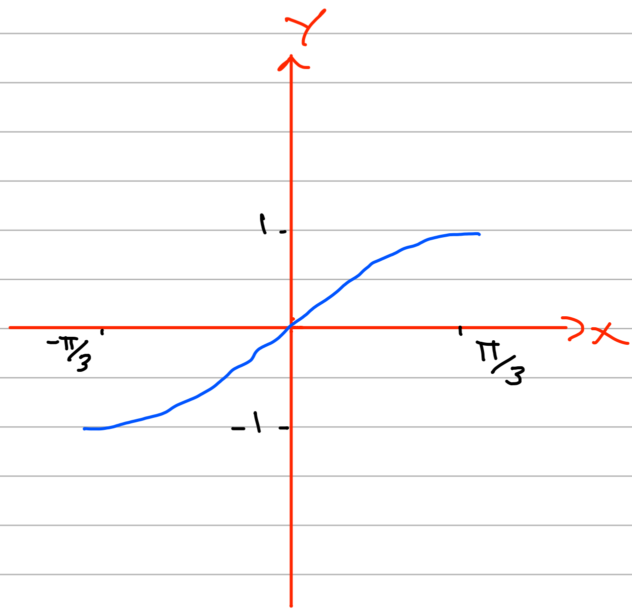
$Y = \sin(-\pi/3) = -\frac{\sqrt{3}}{2}$

A = area of Circle

$= A_1 - A_2 = \frac{1}{4}\pi(1)^2 - \frac{1}{4}\pi(-1)^2$

$= 0$

$A_1 = A_2$



17. In each part, evaluate the integral, given that

$$f(x) = \begin{cases} |x-2|, & x \geq 0 \\ x+2, & x < 0 \end{cases}$$

(a) $\int_{-2}^{0} f(x)\,dx$ (b) $\int_{-2}^{2} f(x)\,dx$

(c) $\int_{0}^{6} f(x)\,dx$ (d) $\int_{-4}^{6} f(x)\,dx$

a) $\int_{-2}^{0} x+2\,dx$

$Y = x+2$

at $x = -2$

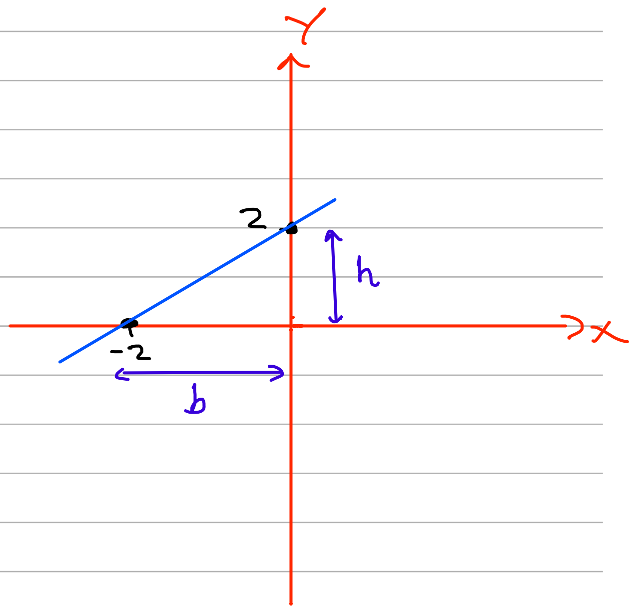
$Y = -2+2 \Rightarrow Y = 0$ $(-2, 0)$

at $x = 0$

$Y = 0+2 \Rightarrow Y = 2$ $(0, 2)$

A = area of triangle

$= \frac{1}{2}(b.h) = \frac{1}{2}(2)(2) = 2$



c) $\int_0^6 f(x)\,dx$

$= \int_0^6 |x-2|\,dx$

$Y = x - 2$

$Y = 0 \Rightarrow 0 = x - 2 \Rightarrow \boxed{x = 2}$

at $x = 0$

$Y = |0-2| \Rightarrow Y = |-2| = 2$ $(0, 2)$

at $x = 6$

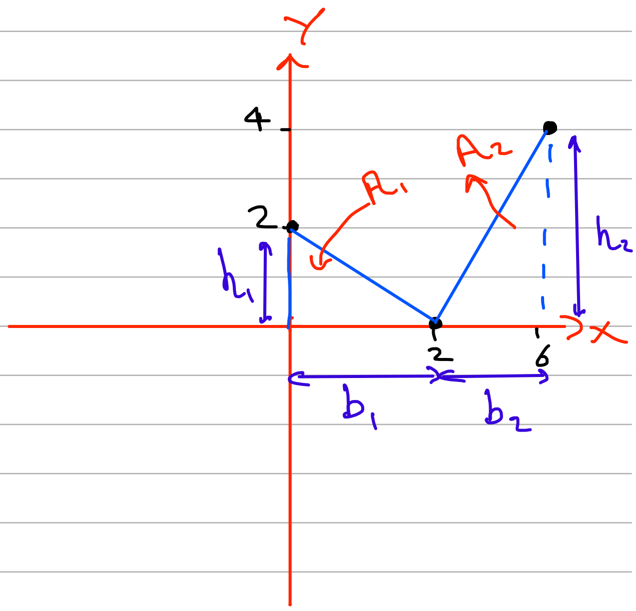
$Y = |6-2| \Rightarrow Y = |4| = 4$ $(6, 4)$

$A = A_1 + A_2$

$= \frac{1}{2}(b_1 \cdot h_1) + \frac{1}{2}(b_2\, h_2)$

$= \frac{1}{2}(2 \cdot 2) + \frac{1}{2}(4 \cdot 4)$

$= 2 + 8 = 10$



d) $\int_{-4}^{6} f(x)\,dx$

$$\int_{-4}^{6} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_{0}^{6} f(x)\,dx$$

$$= \int_{-4}^{0} x+2\,dx + \int_{0}^{6} |x-2|\,dx$$

$$= \int_{-4}^{0} x+2\,dx + 10$$

$y = x+2$

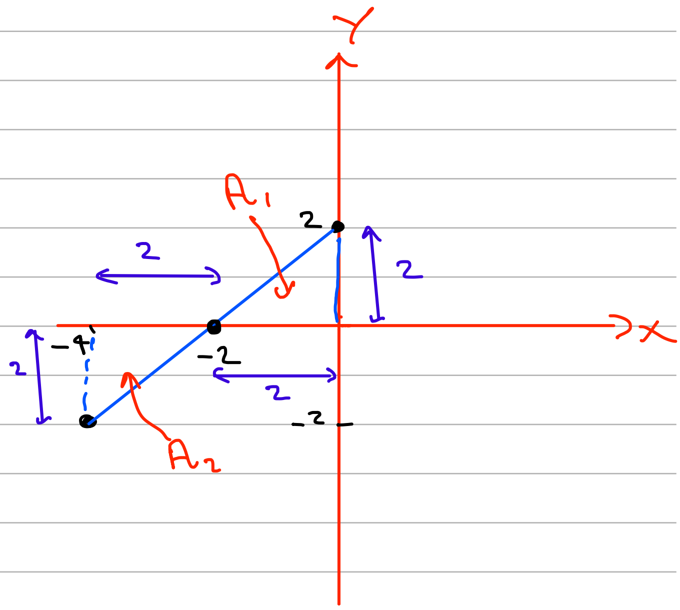
at $x = 0$

$y = 0+2 \Rightarrow y = 2$ $\quad (0, 2)$

at $x = -4$

$y = -4+2 \Rightarrow y = -4$ $\quad (-4, -2)$

$y = 0 \Rightarrow x = -2$



$$A = A_1 + (-A_2) = A_1 - A_2$$

$$= \frac{1}{2} b_1 h_1 - \frac{1}{2} b_2 h_2$$

$$= \frac{1}{2}(2)(2) - \frac{1}{2}(2)(2) = 2 - 2 = 0$$

$$A = 0 + 10 = 10$$

22. Find $\int_1^4 [3f(x) - g(x)]\,dx$ if

$$\int_1^4 f(x)\,dx = 2 \quad \text{and} \quad \int_1^4 g(x)\,dx = 10$$

$$= 3\int_1^4 f(x)\,dx - \int_1^4 g(x)\,dx$$

$$= 3(2) - 10 = 6 - 10 = -4$$

23. Find $\int_1^5 f(x)\,dx$ if

$$\int_0^1 f(x)\,dx = -2 \quad \text{and} \quad \int_0^5 f(x)\,dx = 1$$

$$\int_0^5 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^5 f(x)\,dx$$

$$1 = -2 + \int_1^5 f(x)\,dx$$

$$1 + 2 = \int_1^5 f(x)\,dx$$

$$\int_1^5 f(x)\,dx = 3$$

24. Find $\int_{3}^{-2} f(x)\,dx$ if

$$\int_{-2}^{1} f(x)\,dx = 2 \quad \text{and} \quad \int_{1}^{3} f(x)\,dx = -6$$

$$\int_{3}^{-2} f(x)\,dx = -\int_{-2}^{3} f(x)\,dx$$

$$= -\left[\int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx\right]$$

$$= -(2+(-6)) = -(2-6) = 4$$

33–34 Use Theorem 4.5.6 to determine whether the value of the integral is positive or negative. ■

33. (b) $\int_0^4 \frac{x^2}{3 - \cos x}\, dx$

$x^2 \geq 0$ for all x

$-1 \leq \cos x \leq 1$

$1 \geq -\cos x \geq -1$

$3+1 \geq 3-\cos x \geq -1+3$

$4 \geq 3-\cos x \geq 2$

$\therefore 3-\cos x > 0$ for all x

$f(x) \geq 0$ on $[0, 4]$

$\int_0^4 \frac{x^2}{3-\cos x}\, dx \geq 0$

So integral is Positive