



Princess Nourah Bint  
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# General Physics (2) for Engineering

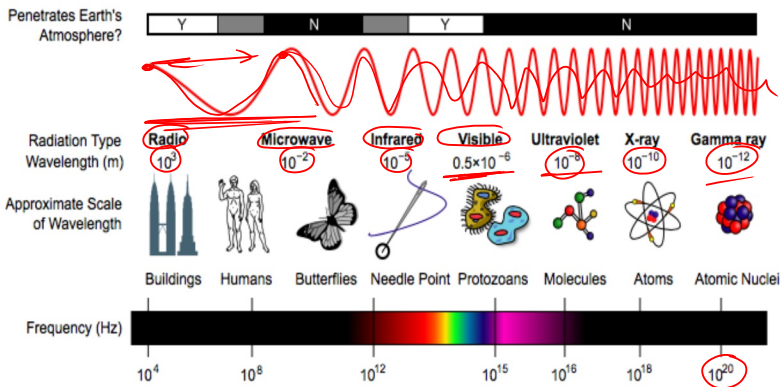
## PHYS 103

# Wave Motion

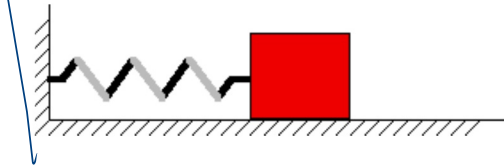
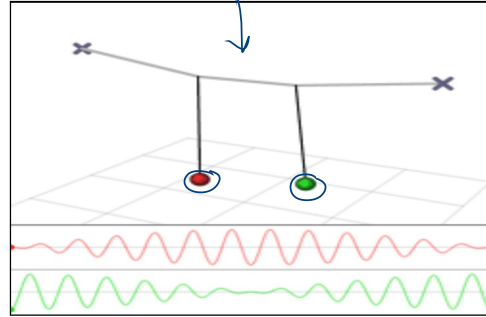
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# Examples of Waves

Electromagnetic waves



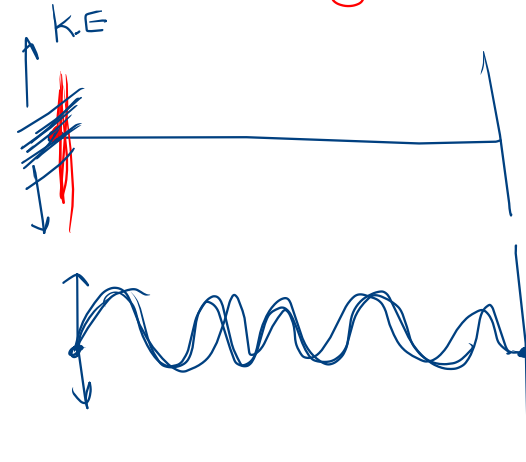
Mechanical waves



Waves

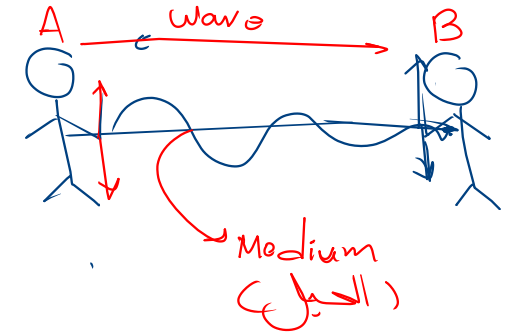
Mechanical waves

تحتاج إلى وسط مادي  
تنتقل خلاله



No Medium  
electromagnetic waves

(Light)  
الضوء



## Propagation of a Disturbance

الطاقة  
بما تنقل  
بدون انتقال الوسط

• A **wave** is a traveling disturbance that transfers energy from one location to another without transporting matter.

• In mechanical waves, elements of the medium oscillate about their equilibrium positions while the wave moves through the medium.

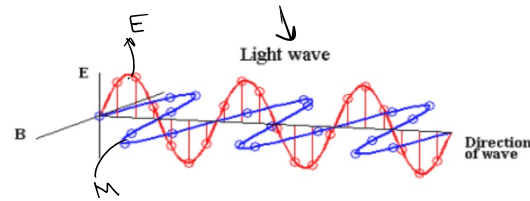
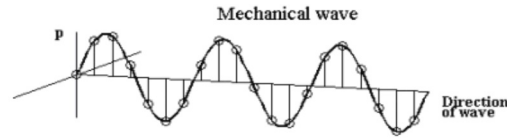
• Wave motion is produced by restoring forces between adjacent elements of the medium.

القوة بين جزيئات الوسط

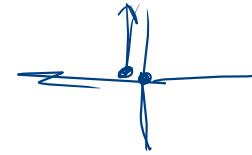
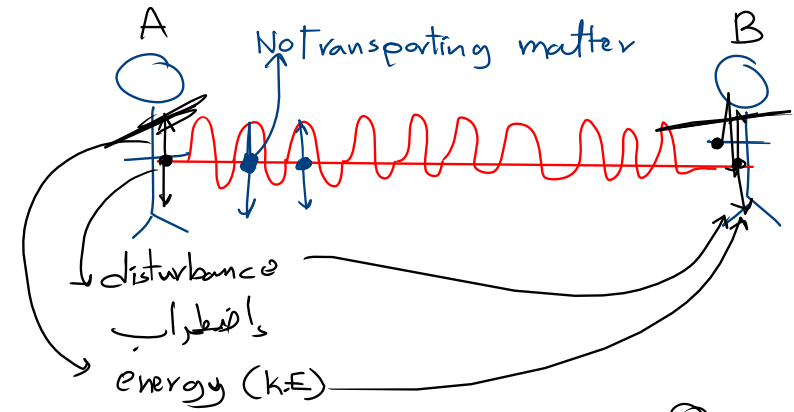
• All **mechanical waves** require:

- (1) some source of disturbance
- (2) a medium that can be disturbed and cannot travel in vacuum
- (3) some physical mechanism through which elements of the medium can influence each other

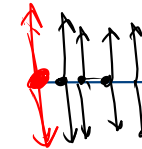
الفراغ



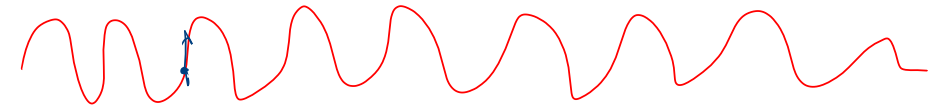
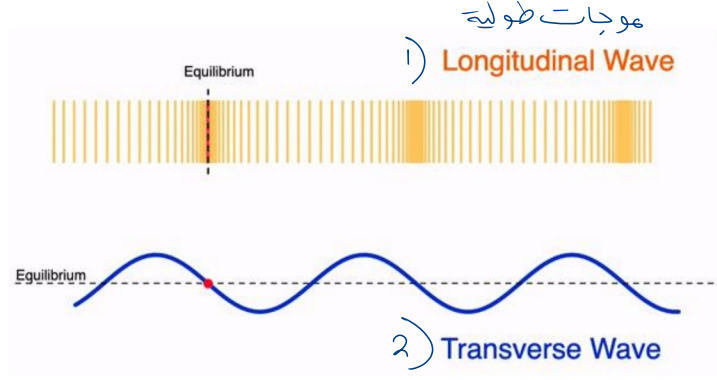
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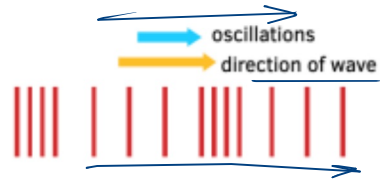
Forces between elements of medium



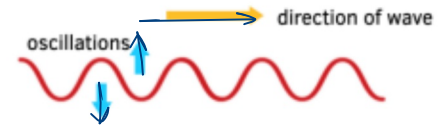
# Types of waves



## Longitudinal Waves



## Transverse Waves



## Types of Mechanical Waves

### 1- Transverse Waves

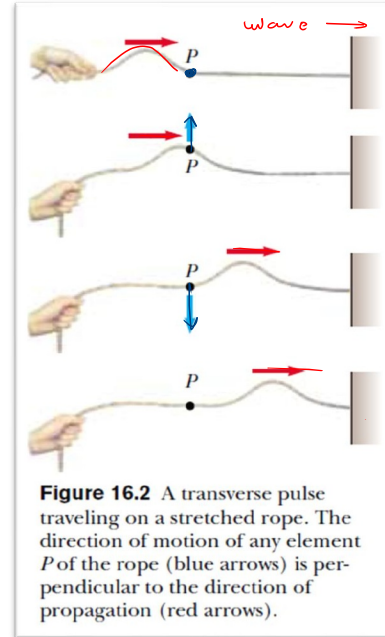


• In a **transverse wave**, the displacement of the medium's elements is **perpendicular** to the direction of wave propagation.

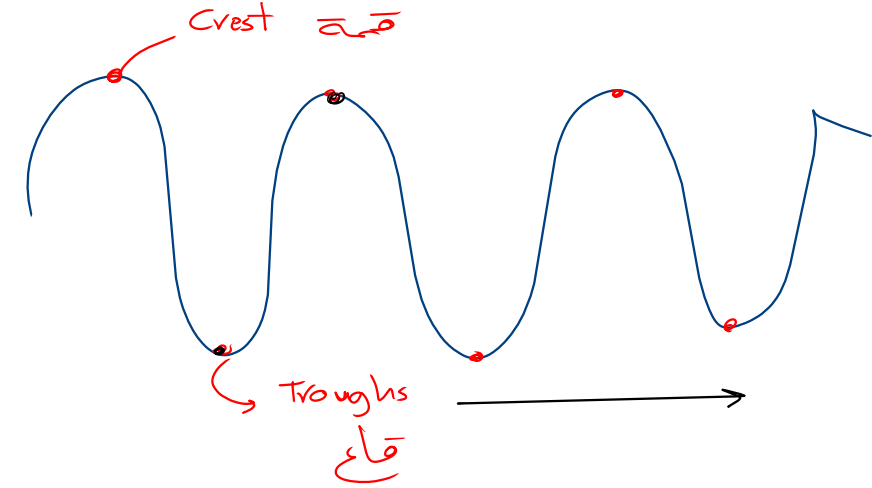
• **Examples**: waves on a stretched string, surface water waves (partially transverse).

• Characterized by **crests** and **troughs**.

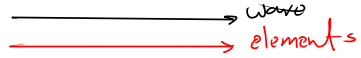
• Energy travels along the medium while elements move up and down.



**Figure 16.2** A transverse pulse traveling on a stretched rope. The direction of motion of any element  $P$  of the rope (blue arrows) is perpendicular to the direction of propagation (red arrows).



## 2- Longitudinal Waves



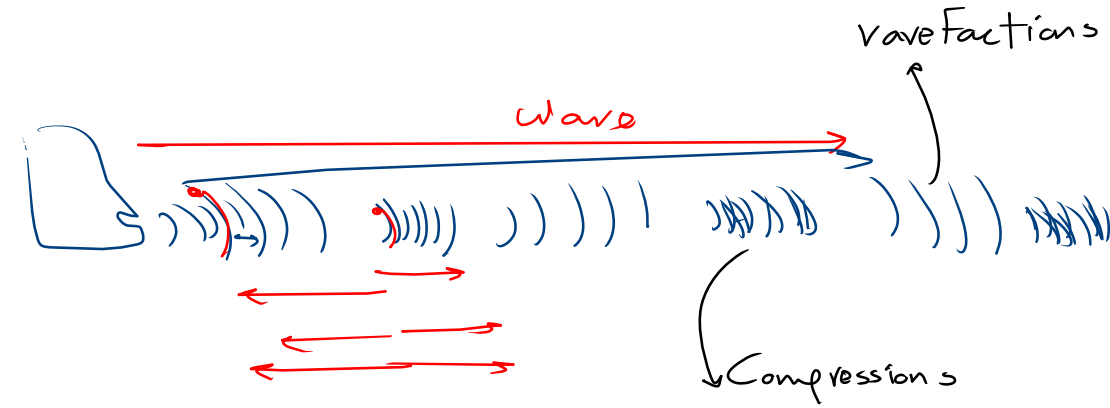
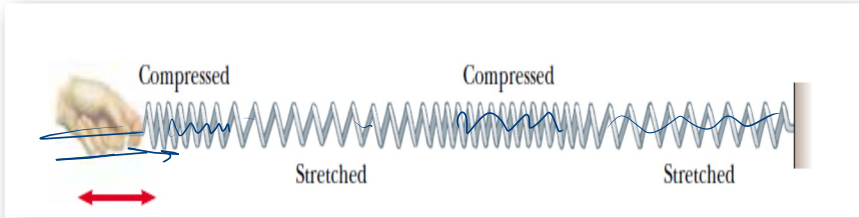
• In a **longitudinal wave**, the displacement of the medium's elements is **parallel** to the direction of wave propagation.

موازى

• The wave consists of alternating regions of:

- **Compressions** (high pressure / high density)
- **Rarefactions** (low pressure / low density)

Sound waves are **longitudinal mechanical waves**





water surface

### 3- Combination of Transverse and Longitudinal Motion (Surface Waves):

Some waves in nature exhibit a **combination of transverse and longitudinal displacements**.

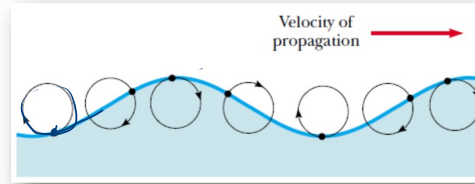
**Surface water waves** are a common example of this type of wave motion.

When a water wave travels on the surface of deep water, elements of water at the surface move in nearly circular paths

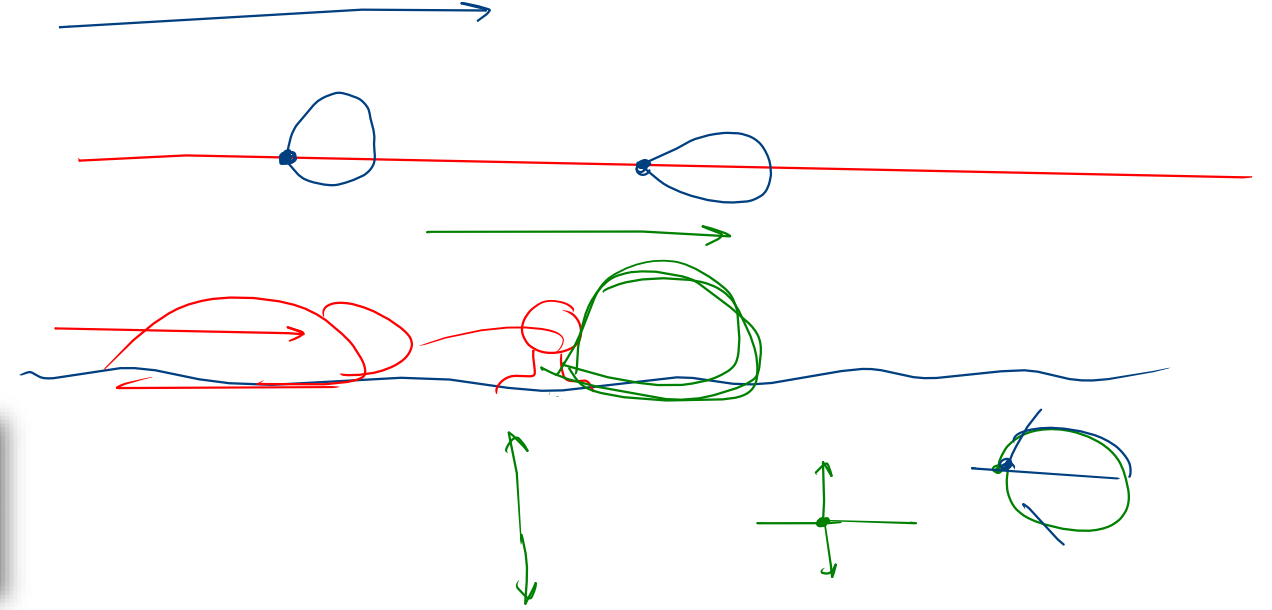
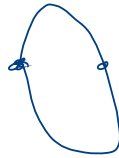
Each element experiences both:

Vertical displacement (transverse motion)

Horizontal displacement (longitudinal motion)

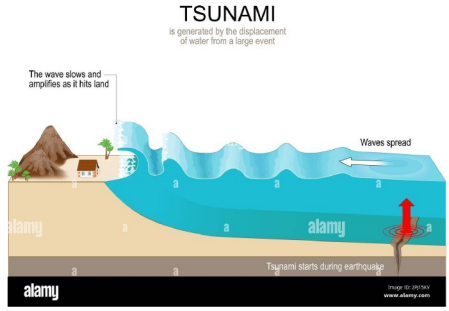
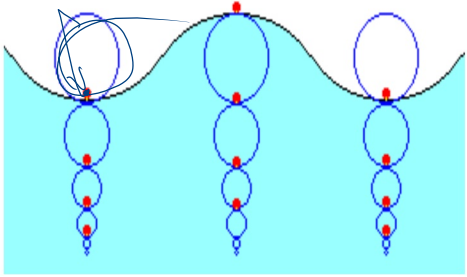


The **transverse displacements** seen in the figure represent the variations in the vertical position of the water elements. The **longitudinal displacement can be explained as follows**: as the wave passes over the water's surface, water elements at the highest points move in the direction of propagation of the wave, whereas elements at the lowest points move in the direction opposite to the propagation.



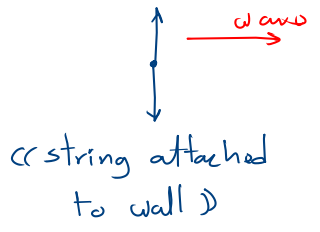
when elements moves in circular motion ((surface waves))

# Examples

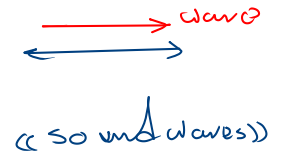


# Mechanical waves

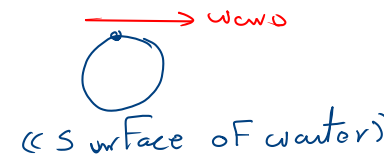
Transverse waves



Longitudinal wave



Surface wave





## Periodic Waves

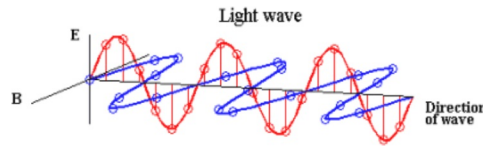
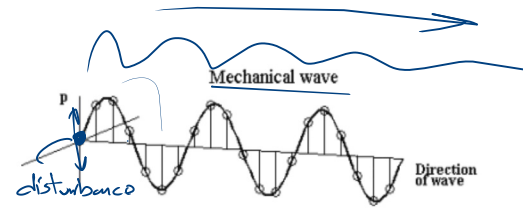
Many waves in nature are **periodic**, meaning the motion of the medium's elements **repeats itself at regular time intervals**.

• A **periodic wave** is produced by a source that vibrates periodically, creating a repeating disturbance in the medium.

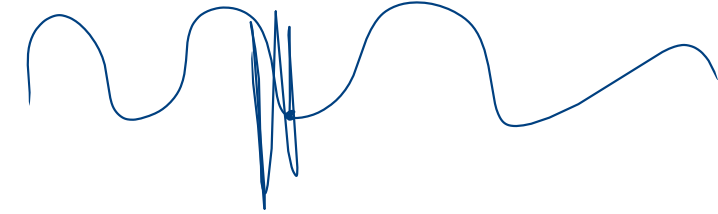
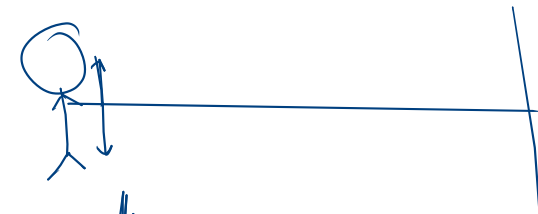
• To describe periodic wave motion **quantitatively**, we focus on a special type of periodic wave.

• The simplest and most important periodic wave is the **sinusoidal wave**

$$2s \rightarrow \begin{matrix} \updownarrow \\ \text{5} \\ 2.5 \end{matrix}$$



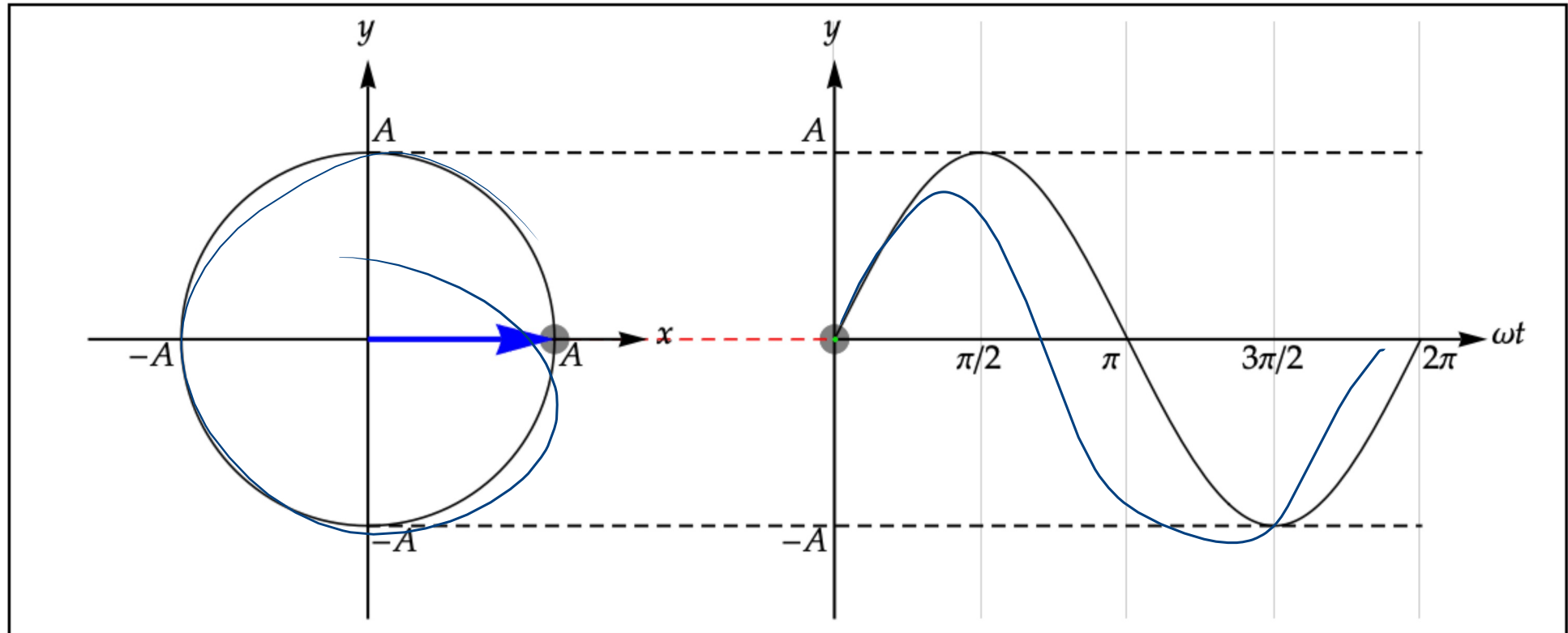
isvr



relation between displacement and position of wave sinusoidal

$y = A \sin(kx - \omega t + \phi)$

# *Wave Function*





## Sinusoidal Waves

• A **sinusoidal wave** is a periodic wave whose displacement varies sinusoidally with position and time.

• Sinusoidal waves are important because **any periodic wave can be represented as a combination of sinusoidal waves.**

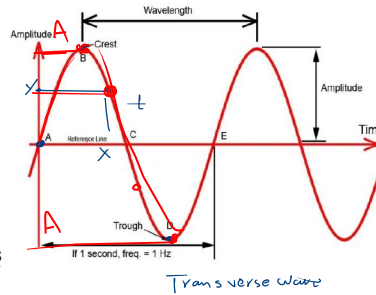
• The general mathematical form of a sinusoidal traveling wave is:

$$y = A \sin(kx - \omega t + \phi)$$

• **Amplitude (A)** represents the maximum displacement of a particle from its equilibrium position.

• The point at which the displacement of the element from its normal position is **highest** is called the **crest**

• The point at which the displacement of the element from its normal position is **lowest** is called the **trough**



displacement

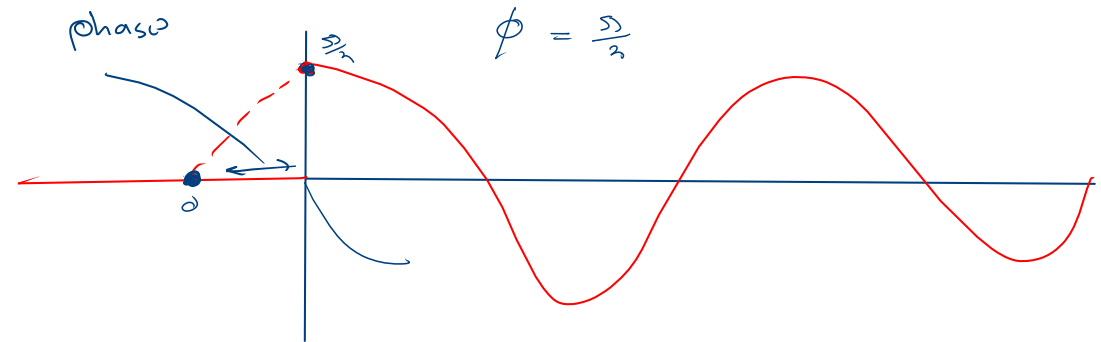
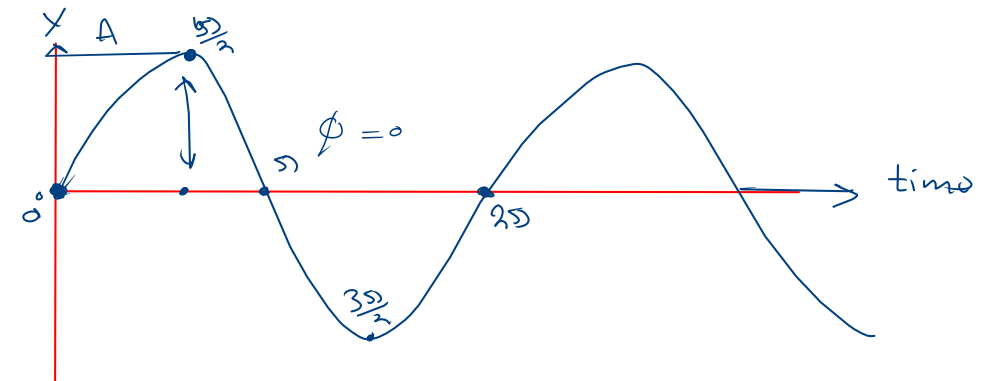
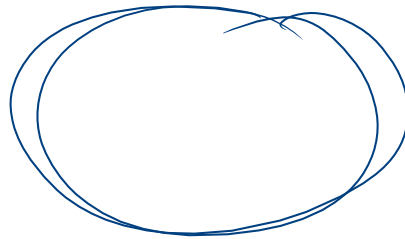
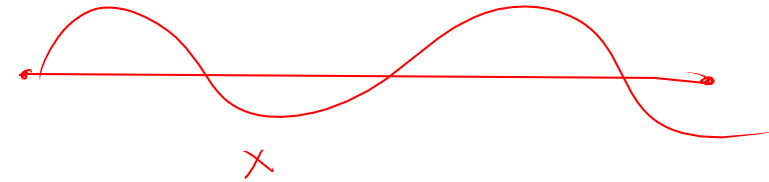
wave number  $k = \frac{2\pi}{\lambda}$

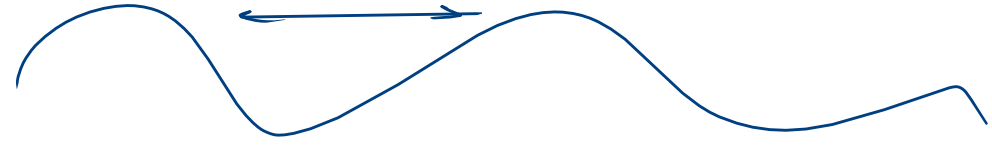
wavelength  $\lambda$

phase constant

Amplitude (Max displacement of element)

$$y = A \sin(kx - \omega t + \phi)$$

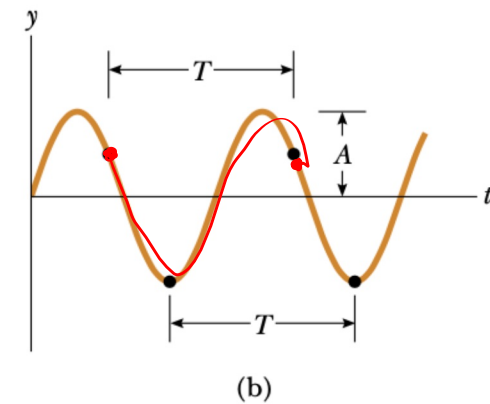
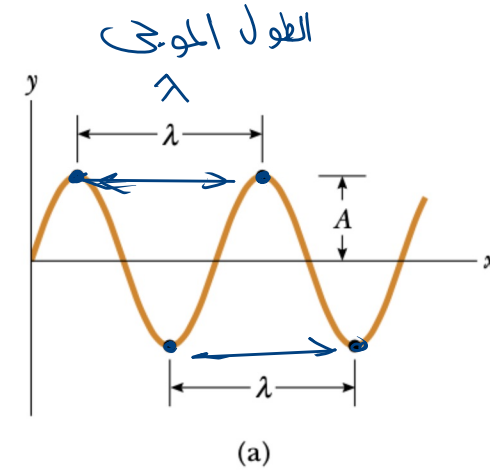
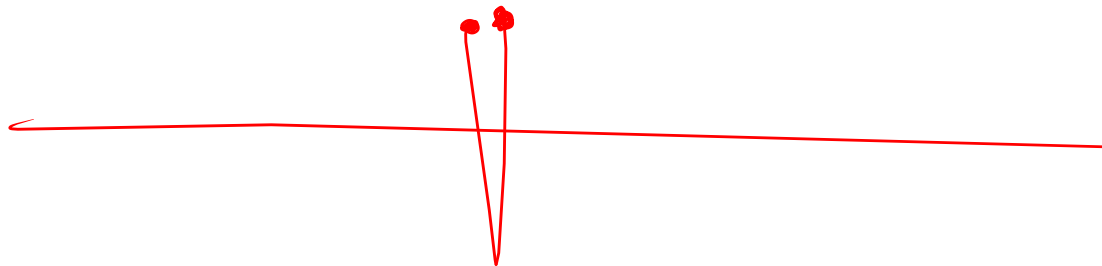
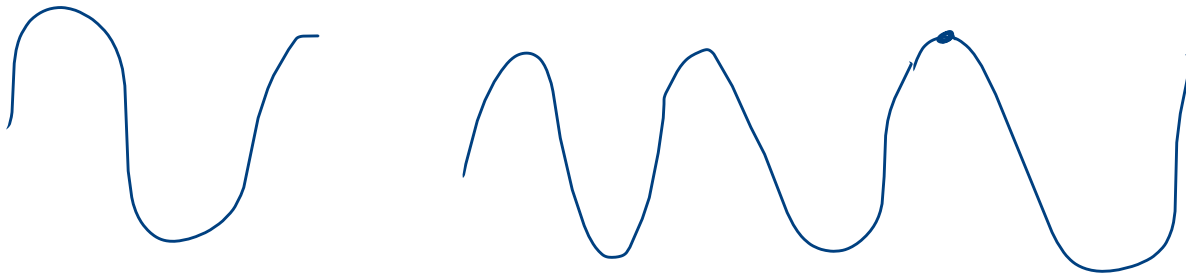


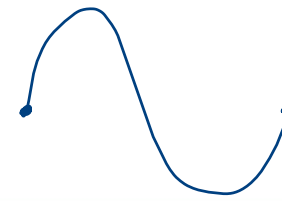
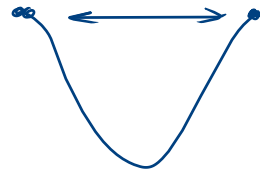


The **phase constant** ( $\phi$ ) is determined uniquely by the position and velocity of the element.

•The **wavelength** ( $\lambda$ ) is the distance between two successive points that are in phase, such as crests or troughs.

•The **period** ( $T$ ) is the time interval required for two identical points (such as the crests) of adjacent waves to pass by a point





in 1 second

•The **frequency (f)** is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval and is related to the period by:

$$f = \frac{1}{T}$$

The most common unit for frequency is **second<sup>-1</sup>**, or **Hertz (Hz)**.

The corresponding unit for  $T$  is seconds.

عدد الدورات في خلال الثانية

$$f = \frac{1}{T} \quad (\text{Hz}) = \frac{1}{\text{period}}$$

The **angular wave number  $k$**  and **angular frequency  $\omega$**  (rad/s) are defined as

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\lambda = \frac{2\pi}{k}$$

$$f = \frac{\omega}{2\pi}$$

The **wave speed ( $v$ )** is given by

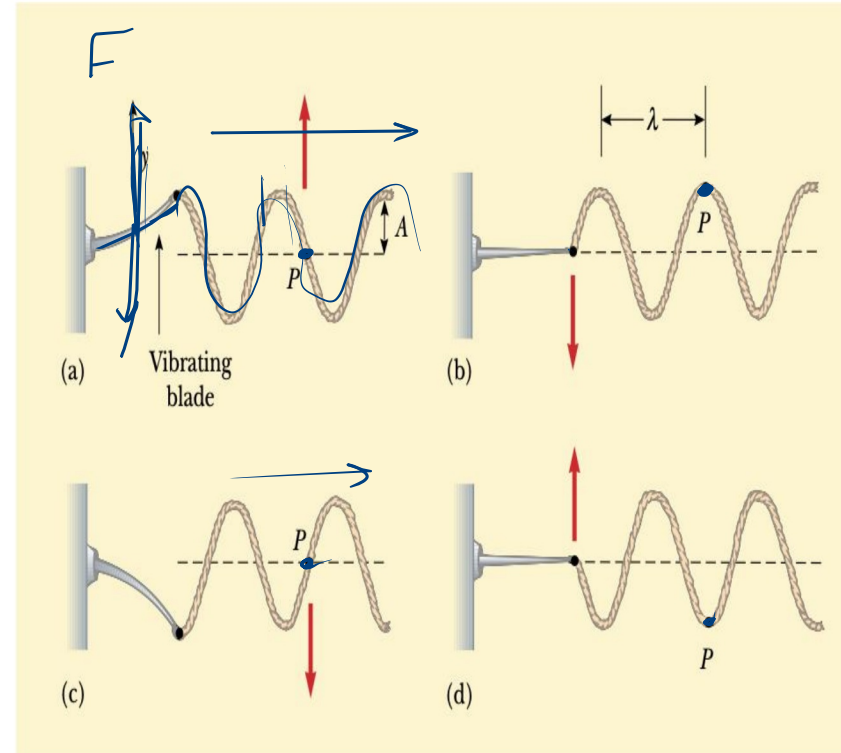
$$v = f\lambda = \cancel{\omega/k}$$

$$v = \lambda f = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

$$v = \lambda f = \frac{\omega}{k} \quad \text{m/s}$$

## Sinusoidal Waves on Strings

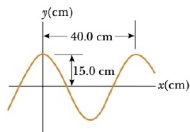
- When the source vibrates in simple harmonic motion (SHM), the resulting wave is a sinusoidal wave.
- Each element of the string oscillates vertically in simple harmonic motion, following the motion of the oscillating blade.
- Every element of the string can be treated as a simple harmonic oscillator with the same frequency as the source.
- Although the string elements oscillate in the **y-direction**, the wave propagates in the **x-direction** with speed  $v$ .
- → This confirms that waves on strings are **transverse waves**.





### Example

A sinusoidal wave traveling in the positive x-direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at  $t=0$  and  $x=0$  is also 15.0 cm



(A) Find the wave number  $k$ , period  $T$ , angular frequency, and speed  $v$  of the wave.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = 0.157 \text{ rad/cm}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = 0.125 \text{ s}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = 50.3 \text{ rad/s}$$

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = 320 \text{ cm/s}$$

(B) Determine the phase constant  $\phi$ , and write a general expression for the wave function

$$y = A \sin(kx - \omega t + \phi)$$

$$15 = 15 \sin(\phi)$$

$$\sin(\phi) = 1 \Rightarrow \phi = \pi/2$$

$$y = A \sin(kx - \omega t + \pi/2) = A \cos(kx - \omega t)$$

at  $t=0, x=0$  ←  
 $y = 15 \text{ cm}$

Givens: -  $A = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$ ,  $\lambda = 40 \times 10^{-2} \text{ m}$ ,  $F = 8 \text{ Hz}$

a)  $k = \frac{2\pi}{\lambda} = \frac{2\pi}{40 \times 10^{-2}} = 15.7 \text{ rad/m}$

$T = \frac{1}{F} = \frac{1}{8} \text{ s} = 0.125 \text{ s}$

$\omega = 2\pi F = 2\pi(8) = 50.3 \text{ rad/s}$

$v = \lambda F = \frac{\omega}{k} = \frac{50.3}{15.7} = 3.2 \text{ m/s}$

b)  $y = A \sin(kx - \omega t + \phi)$

at  $t=0$  and  $x=0$

$$15 = 15 \sin(0 - 0 + \phi)$$

$$15 = 15 \sin \phi \quad \div 15$$

$$1 = \sin \phi$$

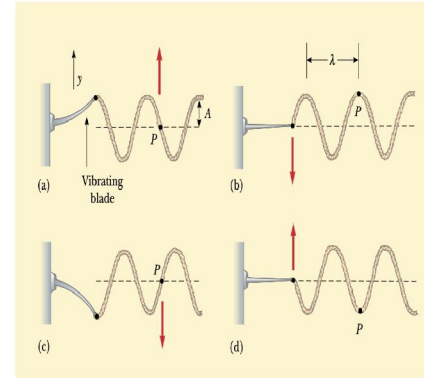
$$\phi = \sin^{-1}(1) = 90^\circ = \frac{\pi}{2}$$

$$y = (15 \times 10^{-2}) \sin(15.7x - 50.3t + \frac{\pi}{2})$$

$$\sin(x + \frac{\pi}{2}) = \cos x$$

### Example

The string shown in the figure is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency and wave number  $k$  for this wave and write an expression for the wave function.



$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00 \text{ Hz}) = 31.4 \text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{31.4 \text{ rad/s}}{20.0 \text{ m/s}} = 1.57 \text{ rad/m}$$

Because  $A = 12.0 \text{ cm} = 0.120 \text{ m}$ , we have

$$y = A \sin(kx - \omega t)$$

$$= (0.120 \text{ m}) \sin(1.57x - 31.4t)$$

Givons :-  $F = 5 \text{ Hz}$  ,  $A = 12 \times 10^{-3} \text{ m}$  ,  $v = 20 \text{ m/s}$

$$- \omega = 2\pi F = 2\pi(5) = 10\pi = \boxed{31.4 \text{ rad/s}}$$

$$- k = \frac{2\pi}{\lambda} \longrightarrow v = \lambda F \quad , \quad v = \frac{\omega}{k} \quad k = \frac{\omega}{v}$$

$$20 = \lambda(5) \quad \lambda = \frac{20}{5} = \boxed{4 \text{ m}}$$

$$k = \frac{2\pi}{4} = \frac{\pi}{2} = \boxed{1.57 \text{ rad/m}}$$

$$\longrightarrow \phi = 0$$

$$y = A \sin(kx - \omega t + \phi)$$

$$y = (12 \times 10^{-3}) \sin(1.57x - 31.4t)$$



The wave function can be written as:

$$y = A \sin(kx - \omega t)$$

For a given element of the string (fixed  $x$ ):

- The element moves only vertically.
- Its horizontal position remains constant.

The transverse velocity and the transverse acceleration of a string element are: :

$$v_y = \left. \frac{dy}{dt} \right|_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A \cos(kx - \omega t)$$

$$a_y = \left. \frac{dv_y}{dt} \right|_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A \sin(kx - \omega t)$$

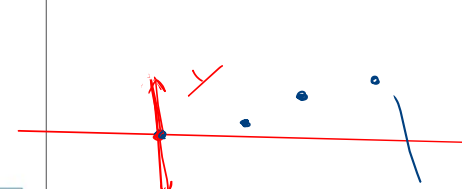
Partial derivatives are used because the displacement  $y$  depends on **both position and time**.

The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y, \max} = \omega A$$

$$a_{y, \max} = \omega^2 A$$

The **wave speed**  $v$  (propagates along the string) is different from the **transverse velocity**  $v_y$  (velocity of a point on the string)



$y = A \sin(kx - \omega t)$   
position — time

$$\frac{d}{dt} \cos \theta = -\sin \theta$$

$$\frac{d}{dt} \sin \theta = \cos \theta$$

$$v_y = \left. \frac{dy}{dt} \right|_{x=\text{constant}} = A(-\omega) \cos(kx - \omega t) = -\omega A \cos(kx - \omega t) \quad (\text{Transverse velocity})$$

$$a_y = \left. \frac{dv_y}{dt} \right|_{x=\text{constant}} = (-\omega A)(-\sin(kx - \omega t))(-\omega) = -\omega^2 A \sin(kx - \omega t) \quad (\text{Transverse acceleration})$$

$$v_{\max} = \omega A$$

$$a_{\max} = \omega^2 A$$

## Superposition Waves

Waves can be combined in the same location in space. To analyze these wave combinations, use the **superposition principle**:

When **two or more traveling waves** move through the same medium, the **resultant displacement** at any point is the **algebraic sum** of the displacements due to the individual waves.

$$y_{\text{resultant}} = y_1 + y_2 + \dots$$

$A \ll \lambda$

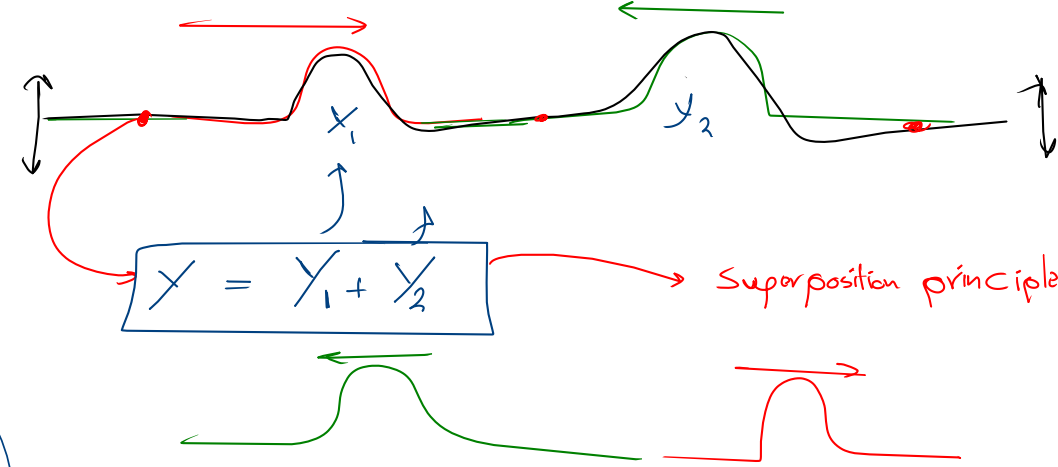
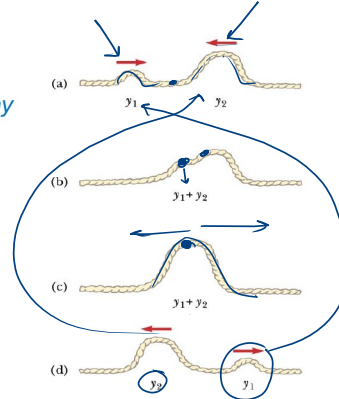
The superposition principle applies only to **linear waves**, where wave amplitudes are small compared to their wavelengths.

بمعنى أن تتغير في الوسط

- Individual waves **pass through each other without being permanently altered**.

- **Interference** is a **direct consequence** of the **superposition principle**. Interference occurs when **two or more waves overlap** in the same region of space at the same time.

- The resulting wave pattern depends on the relative displacements of the overlapping waves.



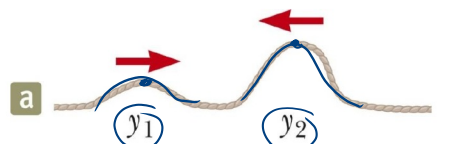


### Constructive Interference

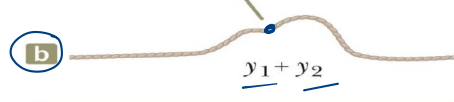


## Superposition Example 1

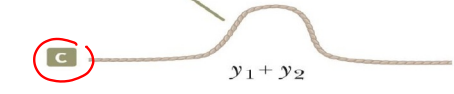
- Two pulses are traveling in opposite directions (a).
  - The wave function of the pulse moving to the right is  $y_1$  and for the one moving to the left is  $y_2$ .   
  $\lambda_1 = \lambda_2, F_1 = F_2$
- The pulses have the same speed but different shapes.  $\rightarrow y_1 \neq y_2$
- The displacement of the elements is positive for both.
- When the waves start to overlap (b), the resultant wave function is  $y_1 + y_2$ .
- When crest meets crest (c) the resultant wave has a larger amplitude than either of the original waves.
- The two pulses separate (d).
  - They continue moving in their original directions.
  - The shapes of the pulses remain unchanged.
- This type of superposition is called **constructive interference**.



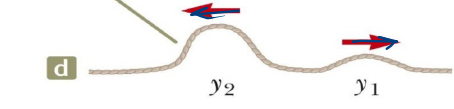
a When the pulses overlap, the wave function is the sum of the individual wave functions.



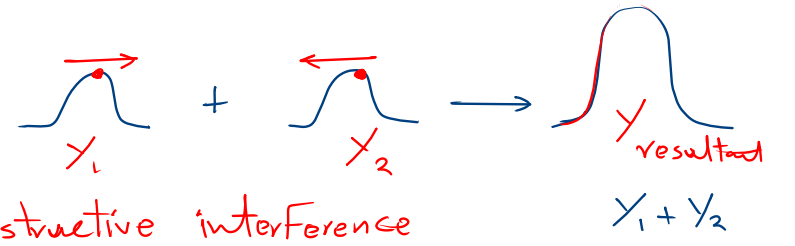
b When the crests of the two pulses align, the amplitude is the sum of the individual amplitudes.



c When the pulses no longer overlap, they have not been permanently affected by the interference.



$$y_{\text{resultant}} = y_1 + y_2 > y_1, y_2$$



$$y_1 \rightarrow +ve, \quad y_2 \rightarrow +ve$$

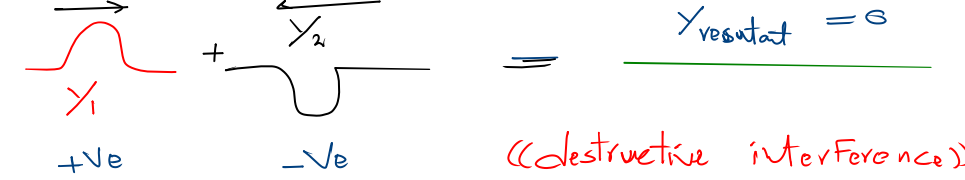
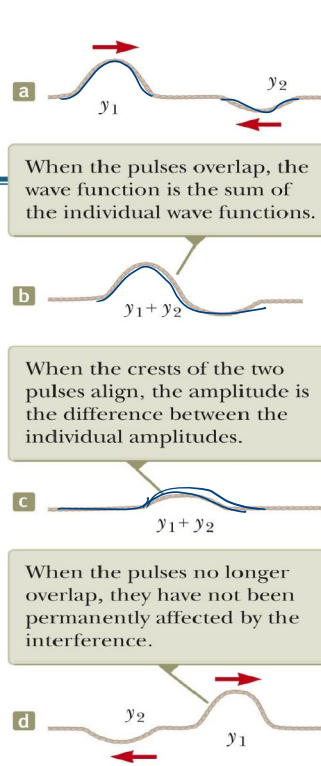
$y_1$  and  $y_2$  should have the same sign



## Superposition Example 2

- Two pulses traveling in opposite directions.
- Their displacements are inverted with respect to each other.
- When these pulses overlap, the resultant pulse is  $y_1 + y_2$ .
- This type of superposition is called **destructive interference**.

### Destructive Interference





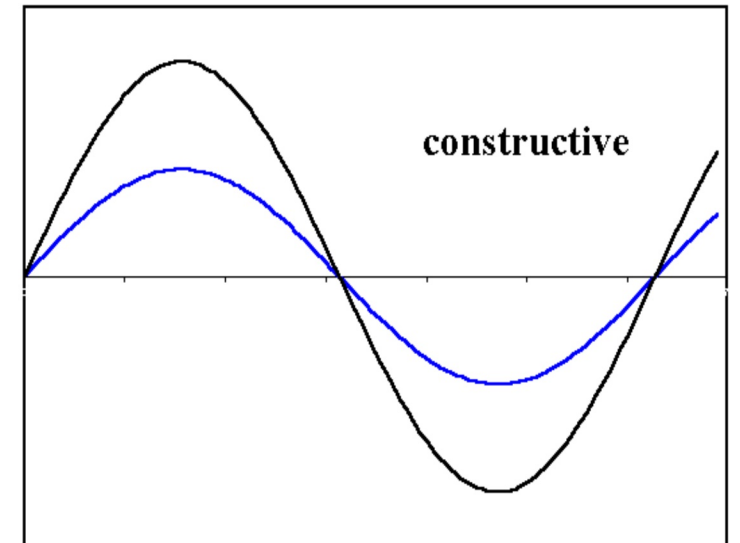
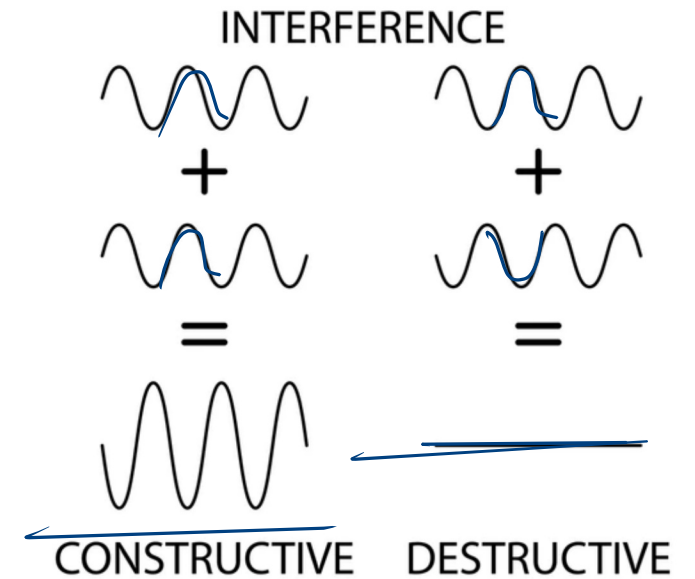
## Types of Interference, Summary

1- **Constructive interference** occurs when the displacements caused by the two pulses are in the same direction.

The amplitude of the resultant pulse is **greater** than either individual pulse.

2- **Destructive interference** occurs when the displacements caused by the two pulses are in opposite directions.

The amplitude of the resultant pulse is **less** than either individual pulse.





## Superposition of Sinusoidal Waves

Assume two waves are traveling in the same direction in a linear medium, with the same frequency, wavelength and amplitude.

The waves differ only in phase:

$$y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx - \omega t + \phi)$$

Hence, the resultant wave function  $y$  is

$$y = y_1 + y_2 = A [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

$$\sin a + \sin b = 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right)$$

$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

$k, \omega$

The resultant wave function,  $y$ , is also sinusoidal.

The resultant wave has the **same frequency and wavelength** as the original waves.

The amplitude of the resultant wave is  **$2A \cos(\phi/2)$** .

The phase constant of the resultant wave is  **$\phi/2$** .

$$Y_1 = A \sin(Kx - \omega t)$$

$$Y_2 = A \sin(Kx - \omega t + \phi)$$

$$Y_{\text{resultant}} = Y_1 + Y_2$$

$$= A \sin(Kx - \omega t) + A \sin(Kx - \omega t + \phi)$$

$$= A (\sin(\underbrace{Kx - \omega t}_b) + \sin(\underbrace{Kx - \omega t + \phi}_a))$$

$$- \sin a + \sin b = 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right)$$

$$Y_{\text{resultant}} = A (2 \cos\left(\frac{\phi}{2}\right) \sin\left(Kx - \omega t + \frac{\phi}{2}\right))$$

$$= \underbrace{2A \cos\left(\frac{\phi}{2}\right)}_X \sin\left(Kx - \omega t + \frac{\phi}{2}\right)$$

$$X = A \sin(Kx - \omega t)$$

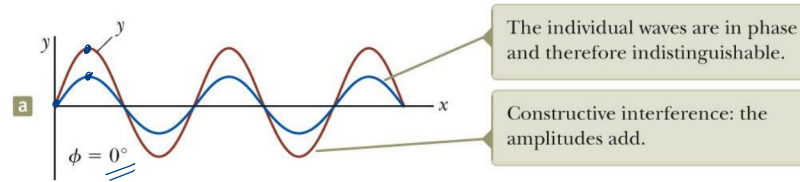
$$\cancel{(Kx - \omega t + \phi)} - \cancel{(Kx - \omega t)} = \phi$$

$$Kx - \omega t + \phi + Kx - \omega t = \frac{2(Kx - \omega t) + \phi}{2} = (Kx - \omega t + \frac{\phi}{2})$$



## Sinusoidal Waves with Constructive Interference

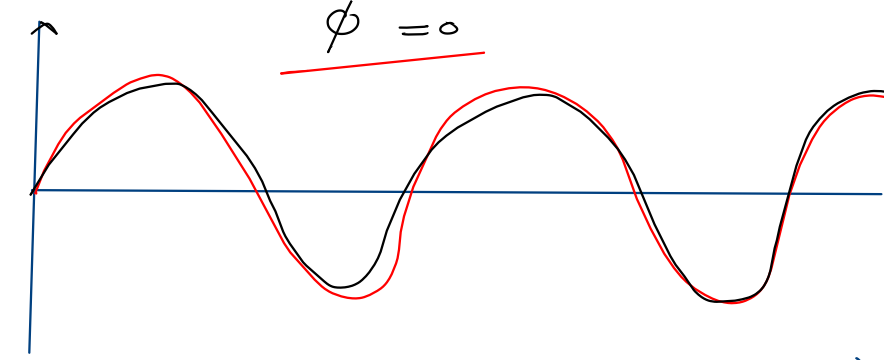
- When  $\phi = 0$ , then  $\cos(\phi/2) = 1$
- The amplitude of the resultant wave is  $2A$ .
  - The crests of the two waves are at the same location in space.
- The waves are everywhere **in phase**.
- The waves interfere constructively.
- In general, constructive interference occurs when  $\cos(\Phi/2) = \pm 1$ .
  - That is, when  $\Phi = 0, 2\pi, 4\pi, \dots$  rad
    - When  $\Phi$  is an even multiple of  $\pi$



$$y_1 \rightarrow A$$

$$y_2 \rightarrow A$$

$$y = 2A$$



$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin(kx - \omega t + \frac{\phi}{2})$$

when  $\phi = 0$

$$y = 2A \cos(0) \sin(kx - \omega t)$$

$$= 2A \sin(kx - \omega t)$$

in case  $\cos\left(\frac{\phi}{2}\right) = \pm 1 \rightarrow$  constructive

$$\phi = 0, 2\pi, 4\pi, 6\pi \text{ (even)}$$

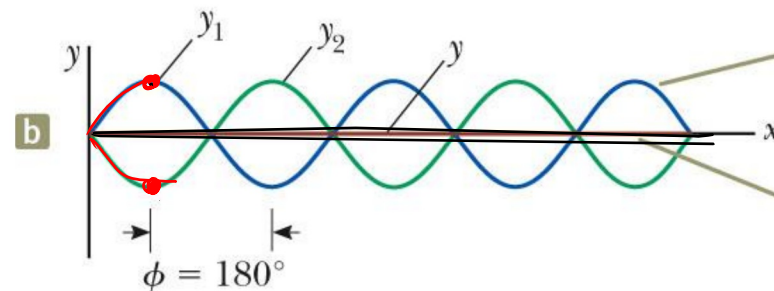
## Sinusoidal Waves with Destructive Interference

- When  $\phi = \pi$ , then  $\cos(\phi/2) = 0$ 
  - Also, any odd multiple of  $\pi$
- The amplitude of the resultant wave is 0.
  - See the straight red-brown line in the figure.
  - The waves are everywhere out of phase.
- The waves interfere destructively.

$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

$$y = \text{Zero} \text{ destructive interference}$$

$$\phi = \pi, 3\pi, 5\pi, 7\pi$$



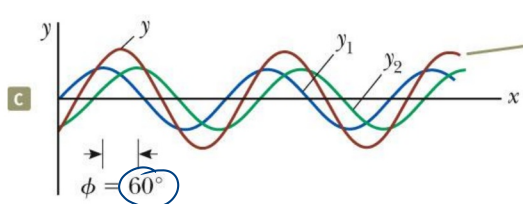
The individual waves are  $180^\circ$  out of phase.

Destructive interference: the waves cancel.

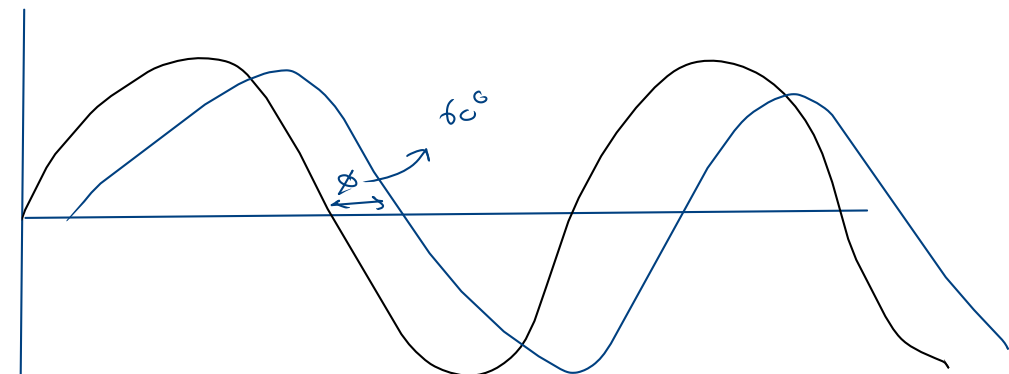
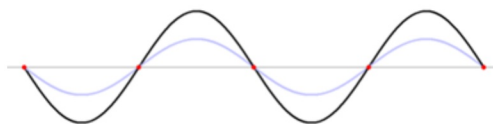


## Sinusoidal Waves, General Interference

- When  $\phi$  is other than 0 or an even multiple of  $\pi$ , the amplitude of the resultant is between 0 and  $2A$ .
- The wave functions still add
- The interference is neither constructive nor destructive.



This intermediate result is neither constructive nor destructive.



$$\cos\left(\frac{\phi}{2}\right) = \cos(30^\circ) =$$

$$y_{\text{result}} = 0$$

$$y_{\text{result}} = 2A$$



## Example

1. Two harmonic waves are described by

$$y_1 = 3 \sin(4x - 700t)$$

$$y_2 = 3 \sin(4x - 700t - 2)$$

What is the amplitude of the resultant wave?

In general, the amplitude of the resultant wave when the two waves with constant phase difference  $\phi$  are superimposed one another is expressed as:  
The two waves are:

$$y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx - \omega t \pm \phi)$$

$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t \pm \frac{\phi}{2}\right)$$

$$A = 3, \phi = 2 \text{ rad} = 114.59114.59^\circ$$

The resultant Amplitude =

$$2A \cos\left(\frac{\phi}{2}\right) = (2)(3) \cos\left(\frac{114.59}{2}\right) = 3.24 \text{ m}$$

$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin(kx - \omega t + \frac{\phi}{2})$$

$$x_t \rightarrow A = 3, \quad k = 4, \quad \omega = 700 \rightarrow \text{degree لازم تكون}$$

$$\rightarrow \phi = 2 \text{ rad} \quad \times \frac{180}{57} = \frac{2 \times 180}{57} = 114.59^\circ = \phi$$

radian  $\times \frac{180}{57} \rightarrow$  degree

$$y_{\text{resultant}} \rightarrow 2A \cos\left(\frac{\phi}{2}\right) = 2(3) \cos\left(\frac{114.59}{2}\right) = \boxed{3.24 \text{ m}}$$

### Standing Waves

Assume two waves with the same amplitude, frequency and wavelength, traveling in opposite directions in a medium.

$$y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx + \omega t)$$

The waves combine in accordance with the waves in interference model.

They interfere according to the superposition principle.

The resultant wave will be:

$$y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

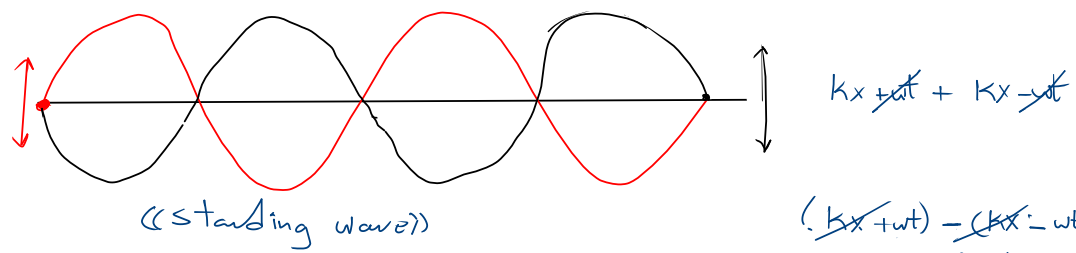
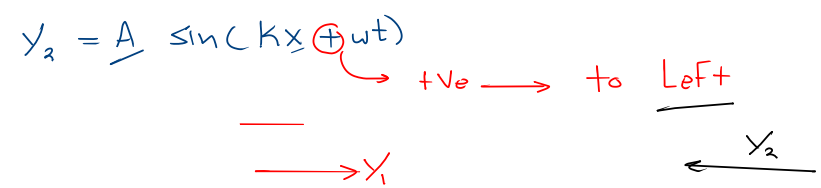
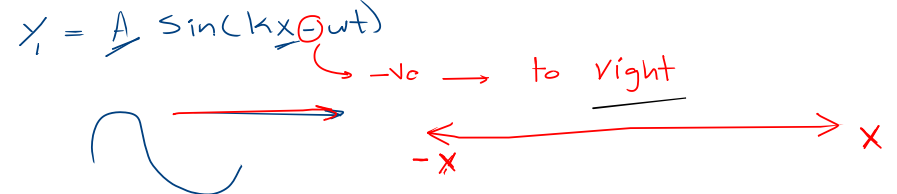
$$y = (2A \sin kx) \cos \omega t$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

This is the wave function of a standing wave.

- There is no  $kx - \omega t$  term, and therefore it is not a traveling wave.

In observing a standing wave, there is no sense of motion in the direction of propagation of either of the original waves.



$$y_1 = A \sin(kx - \omega t) \quad , \quad y_2 = A \sin(kx + \omega t)$$

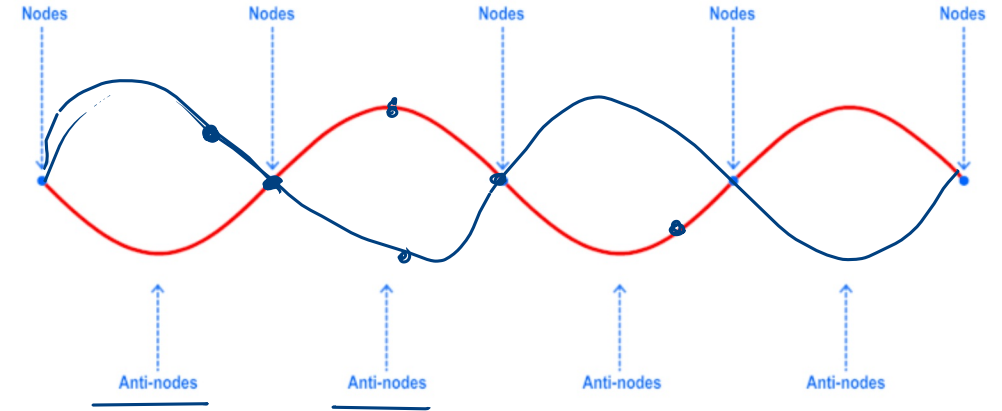
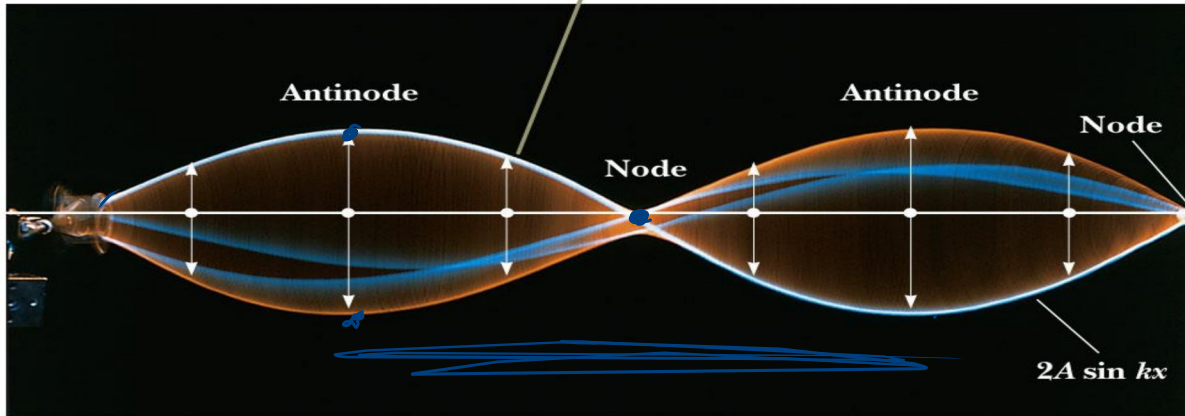
$$y_{\text{resultant}} = A \sin(kx - \omega t) + A \sin(kx + \omega t) = A (\sin(\underbrace{kx - \omega t}_b) + \sin(\underbrace{kx + \omega t}_a))$$

$$= A (2 \cos(\frac{2\omega t}{2}) \sin(\frac{2kx}{2}))$$

$$y = \underbrace{2A \sin(kx)}_{\text{Amplitude}} \cos(\omega t)$$

There is no  $kx$  <sup>position</sup>  
 wave not travelling  
 ((standing wave))

The amplitude of the vertical oscillation of any element of the string depends on the horizontal position of the element. Each element vibrates within the confines of the envelope function  $2A \sin kx$ .



- Note the stationary outline that results from the superposition of two identical waves traveling in opposite directions.

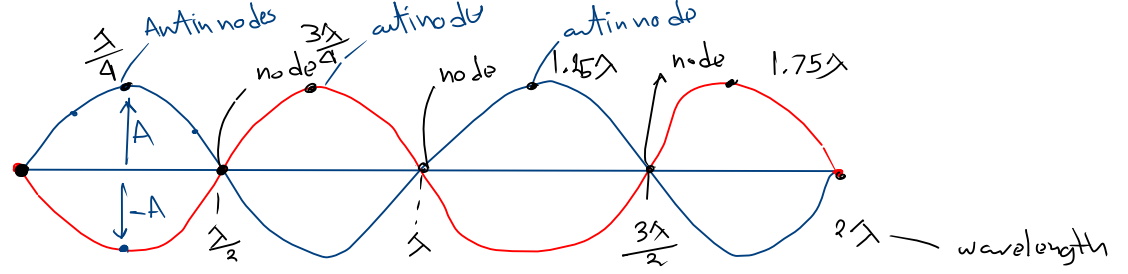
- The amplitude of the simple harmonic motion of a given element is  $2A \sin kx$ .

- This depends on the location  $x$  of the element in the medium.

- Each individual element vibrates at  $\omega$ .  $\rightarrow$  Frequency of The Two waves



$$y_1 = A, y_2 = A$$



There are three types of amplitudes used in describing waves.

- The amplitude of the individual waves, A
- The amplitude of the simple harmonic motion of the elements in the medium, 2A sin kx
  - A given element in the standing wave vibrates within the constraints of the envelope function 2A sin kx.
- The amplitude of the standing wave, 2A

A node occurs at a point of zero amplitude.

- These correspond to positions of x where,  $\sin kx = 0$   $kx = \pi, 2\pi, 3\pi, \dots$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots = \left(\frac{n\lambda}{2}\right) n = 0, 1, 2, 3, \dots$$
 ← Positions of nodes

An antinode occurs at a point of maximum displacement, 2A.

- These correspond to positions of x where,  $\sin kx = \pm 1$   $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots = \frac{n\lambda}{4} n = 1, 3, 5, \dots$$
 ← Positions of antinodes  
odd numbers

$$y_{\text{resultant}} = 2A \sin(kx)$$

at node

$$y_{\text{resultant}} = (2A \sin kx = 0)$$

$$\sin kx = 0$$
$$kx = \sin^{-1}(0) = \pi, 2\pi, 3\pi, \dots$$

~~2pi~~  $x = \frac{n\lambda}{2}, n = 0, 1, 2, 3$

position of nodes

$x = \frac{n\lambda}{2}, n = 0, 1, 2, 3$

at antinodes

$$y_{\text{resultant}} = 2A \sin kx = 2A$$

$$\sin kx = 1$$
$$kx = \sin^{-1}(1) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$kx = \frac{n\pi}{2}, n = 1, 3, 5, 7$$

~~3pi~~  $\cdot x = \frac{n\lambda}{4}$

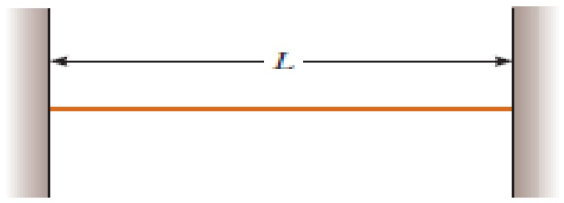
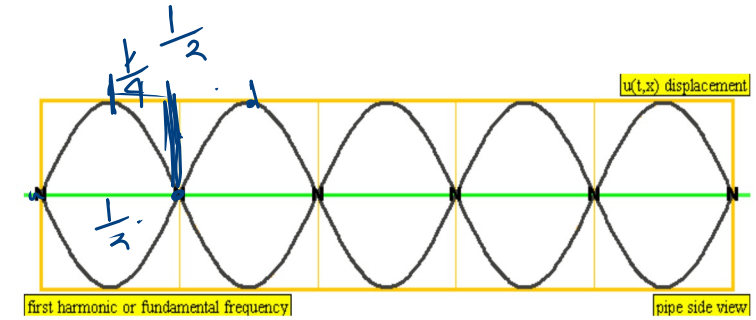
$x = \frac{n\lambda}{4}, n = 1, 3, 5, 7$

position of antinodes

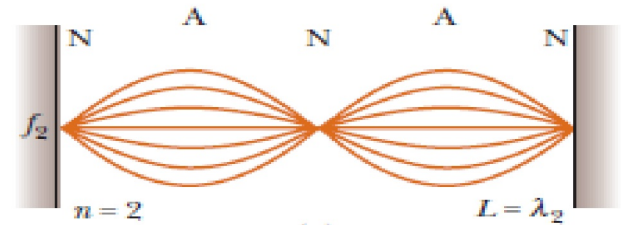


## Features of Nodes and Antinodes

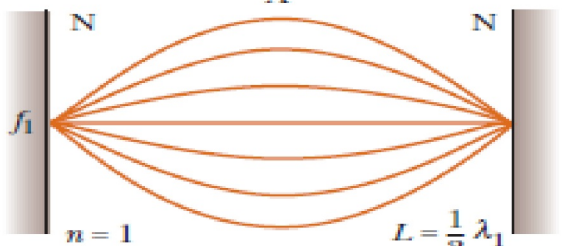
The distance between adjacent antinodes is equal to  $\lambda/2$ .  
The distance between adjacent nodes is equal to  $\lambda/2$ .  
The distance between a node and an adjacent antinode is  $\lambda/4$ .



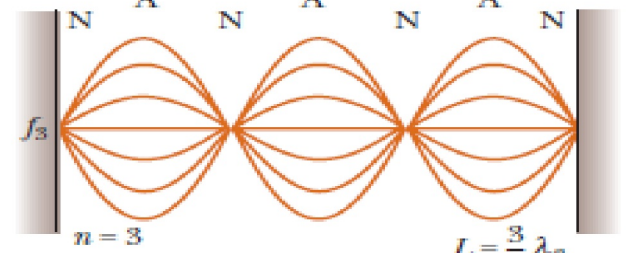
(a)



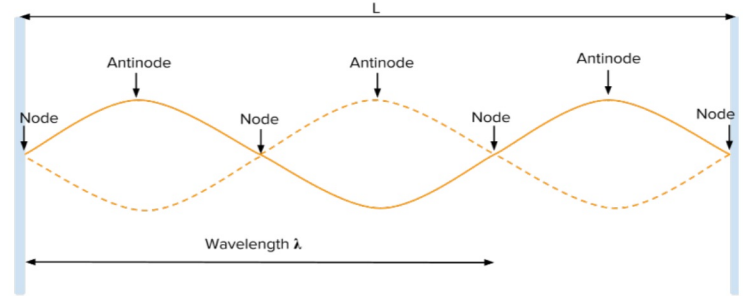
(c)



(b)



(d)





## Example

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = 4.0 \sin(3.0x - 2.0t) \quad \rightarrow$$

$$y_2 = 4.0 \sin(3.0x + 2.0t) \quad \leftarrow$$

where  $x$  and  $y$  are measured in centimeters and  $t$  is in seconds.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at  $x = 2.3$  cm.

From the equations for the waves, we see that  $A=4.0$  cm,  $k=3.0$  rad/cm, and  $\omega=2.0$  rad/s.

The resultant standing wave will be:

$$y = (2A \sin kx) \cos \omega t$$

Resultant amplitude

$$y = (2A \sin kx) \cos \omega t = 8.0 \sin 3.0x \cos 2.0t$$

$$y_{\max} = (8.0 \text{ cm}) \sin 3.0x \Big|_{x=2.3}$$

$$= (8.0 \text{ cm}) \sin (6.9 \text{ rad}) = 4.6 \text{ cm}$$

$$a) \quad y_{\text{resultant}} = \underline{2A \sin kx} \cos(\omega t)$$

$$\text{Amplitude } 2A \sin kx \quad \text{at } x = 2.3 \text{ cm}$$

$$= 2(4) \sin(3(2.3))$$

$$= 8 \sin(6.9 \text{ rad}) = \boxed{4.6 \text{ cm}}$$

put calculator on Radian  
Rad



(B) Find the positions of the nodes and antinodes if one end of the string is at  $x = 0$ .

Find the wavelength of the traveling waves:

$$k = \frac{2\pi}{\lambda} = 3.0 \text{ rad/cm} \rightarrow \lambda = \frac{2\pi}{3.0} \text{ cm}$$

to find the locations of the nodes:

$$x = n \frac{\lambda}{2} = n \left( \frac{\pi}{3.0} \right) \text{ cm} \quad n = 0, 1, 2, 3, \dots$$

to find the locations of the antinodes:

$$x = n \frac{\lambda}{4} = n \left( \frac{\pi}{6.0} \right) \text{ cm} \quad n = 1, 3, 5, 7, \dots$$

Nodes  $\rightarrow x = \frac{n\lambda}{2}, \quad n = 0, 1, 2, 3$

$\rightarrow k = 3 = \frac{2\pi}{\lambda}$ ,  $\lambda = \frac{2\pi}{3} \text{ cm}$

$x = \frac{n(\frac{2\pi}{3})}{2} = \frac{n\pi}{3}, \quad n = 0, 1, 2, 3 \text{ cm}$

antinodes  $\rightarrow x = \frac{n\lambda}{4}, \quad n = 1, 3, 5, 7$

$= \frac{n(\frac{2\pi}{3})}{4} = \frac{n\pi}{6}, \quad n = 1, 3, 5, 7 \text{ cm}$



## Homework

1- A sinusoidal wave is described by

$$y = (0.25 \text{ m}) \sin(0.30x - 40t)$$

where  $x$  and  $y$  are in meters and  $t$  is in seconds. Determine for this wave the (a) amplitude, (b) angular frequency, (c) angular wave number, (d) wavelength, (e) wave speed, and (f) direction of motion.

2- Two waves in one string are described by the wave functions

$$y_1 = 3.0 \cos(4.0x - 1.6t)$$

and

$$y_2 = 4.0 \sin(5.0x - 2.0t)$$

where  $y$  and  $x$  are in centimeters and  $t$  is in seconds. Find the superposition of the waves  $y_1 + y_2$  at the points (a)  $x = 1.00$ ,  $t = 1.00$ , (b)  $x = 1.00$ ,  $t = 0.500$ , and (c)  $x = 0.500$ ,  $t = 0$ . (Remember that the arguments of the trigonometric functions are in radians.)