IQ ACADEMY

Exercise set (2.4)

1. Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

a) a_0 **b)** a_1 **c)** a_4 **d)** a_5

a) $a_0 = 2(-3)^0 + (5)^0 = 2 + 1 = 3$

b) $a_1 = 2(-3)^1 + (5)^1 = -6 + 5 = -1$

c) $a_4 = 2(-3)^4 + (5)^4 = 162 + 625 = 787$

d) $a_5 = 2(-3)^5 + (5)^5 = -486 + 3125 = 2639$

2. What is the term a_8 of the sequence $\{a_n\}$ if a_n equals

a) 2^{n-1}? **b)** 7?

c) $1 + (-1)^n$? **d)** $-(-2)^n$?

a) $a_8 = 2^{8-1} = 2^7 = 128$

b) $a_8 = 7$

c) $a_8 = 1 + (-1)^8 = 2$

d) $a_8 = -(-2)^8 = -256$

4. What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

a) $(-2)^n$? **b)** 3?

c) $7 + 4^n$? **d)** $2^n + (-2)^n$?

a) $a_0 = (-2)^0 = 1$

$a_1 = (-2)^1 = -2$

$a_2 = (-2)^2 = 4$

$a_3 = (-2)^3 = -8$

b) $a_0 = 3$

$a_1 = 3$

$a_2 = 3$

$a_3 = 3$

c) $a_n = 7 + 4^n$

$a_0 = 7 + 4^0 = 8$

$a_1 = 7 + 4^1 = 11$

$a_2 = 7 + 4^2 = 7 + 16 = 23$

$a_3 = 7 + 4^3 = 7 + 64 = 71$

d) $a_n = 2^n + (-2)^n$

$a_0 = 2^0 + (-2)^0 = 1 + 1 = 2$

$a_1 = 2^1 + (-2)^1 = 2 - 2 = 0$

$a_2 = 2^2 + (-2)^2 = 4 + 4 = 8$

$a_3 = 2^3 + (-2)^3 = 8 - 8 = 0$

5. List the first 10 terms of each of these sequences.

a) the sequence that begins with 2 and in which each successive term is 3 more than the preceding term

b) the sequence that lists each positive integer three times, in increasing order

c) the sequence that lists the odd positive integers in increasing order, listing each odd integer twice

d) the sequence whose nth term is $n! - 2^n$

e) the sequence that begins with 3, where each succeeding term is twice the preceding term

f) the sequence whose first term is 2, second term is 4, and each succeeding term is the sum of the two preceding terms

g) the sequence whose nth term is the number of bits in the binary expansion of the number n (defined in Section 4.2)

h) the sequence where the nth term is the number of letters in the English word for the index n

a) 2, 5, 8, 11, 14, 17, 20, 23, 26, 29

b) 1, 1, 1, 2, 2, 2, 3, 3, 3, 4

c) 1, 1, 3, 3, 5, 5, 7, 7, 9, 9

d) $0!-2^0, 1!-2^1, 2!-2^2, 3!-2^3, 4!-2^4, 5!-2^5, 6!-2^6,$

$7!-2^7, 8!-2^8, 9!-2^9$

0, -1, -2, -2, 8, -88, 656, 4912, 40064, 362368

e) 3, 6, 12, 24, 48, 96, 192, 384, 768, 1936

f) 2, 4, 6, 10, 16, 26, 42, 68, 110, 178

g) 1, 2, 2, 3, 3, 3, 3, 4, 4, 4

	binary	
1	1	1
2	10	2
3	11	2
4	100	3
5	101	3
6	110	3
7	111	3
8	1000	4
9	1001	4
10	1010	4

h) 3, 3, 5, 4, 4, 3, 5, 5, 4, 3

one	3
two	3
three	5
four	4
five	4
six	3
seven	5
eight	5
nine	4
ten	3

6. List the first 10 terms of each of these sequences.

a) the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term

b) the sequence whose nth term is the sum of the first n positive integers

c) the sequence whose nth term is $3^n - 2^n$

a) $10, 7, 4, 1, -2, -5, -8, -11, -14, -17$

b) $a_n = \frac{n(n+1)}{2}$

$a_1 = \frac{1(1+1)}{2} = 1$

$a_2 = 3$

$a_3 = 6$

$a_4 = 10$

$a_5 = 15$

$a_6 = 21$

$a_7 = 28$

$a_8 = 36$

$a_9 = 45$

$a_{10} = 55$

c) $a_n = 3^n - 2^n$

$a_1 = 3^1 - 2^1 = 1$

$a_2 = 3^2 - 2^2 = 5$

$a_3 = 3^3 - 2^3 = 19$

$a_4 = 3^4 - 2^4 = 65$

$a_5 = 3^5 - 2^5 = 211$

$a_6 = 3^6 - 2^6 = 665$

$a_7 = 3^7 - 2^7 = 2059$

$a_8 = 3^8 - 2^8 = 6305$

$a_9 = 3^9 - 2^9 = 19171$

$a_{10} = 3^{10} - 2^{10} = 58025$

10. Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -2a_{n-1}$, $a_0 = -1$
b) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$
c) $a_n = 3a_{n-1}^2$, $a_0 = 1$
d) $a_n = na_{n-1} + a_{n-2}^2$, $a_0 = -1$, $a_1 = 0$
e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

a)

n	a_n
0	-1
1	$2a_{1-1} = -2(-1) = 2$
2	$2a_{2-1} = -2(2) = -4$
3	$2a_{3-1} = -2(4) = 8$
4	$2a_{4-1} = -2(8) = -16$
5	$2a_{5-1} = -2(-16) = 32$

b)

n	$a_n = a_{n-1} - a_{n-2}$
0	$a_0 = 2$
1	$a_1 = -1$
2	$a_2 = a_{2-1} - a_{2-2} = -1 - 2 = -3$
3	$a_3 = a_{3-1} - a_{3-2} = -3 - (-1) = -2$
4	$a_4 = a_{4-1} - a_{4-2} = -2 - (-3) = 1$
5	$a_5 = a_{5-1} - a_{5-2} = 1 - (-2) = 3$

c) $a_n = 3a_{n-1}^2 \quad , a_0 = 1$

n	$a_n = a_{n-1}^2$
0	$a_0 = 1$
1	$a_1 = 3a_{1-1}^2 = 3$
2	$a_2 = 3a_{2-1}^2 = 3(9) = 27$
3	$a_3 = 3a_{3-1}^2 = 3(27)^2 = 2187$
4	$a_4 = 3a_{4-1}^2 = 3(2187)^2 = 143489$
5	$a_5 = 3a_{5-1}^2 = 3(143489)^2 = 6.1767 * 10^{10}$

d) $a_n = na_{n-1} + a_{n-2}^2 \quad , a_0 = -1, \; a_1 = 0$

n	$a_n = na_{n-1} + a_{n-2}^2$
0	$a_0 = -1$
1	$a_1 = 0$
2	$a_2 = 2a_{2-1} + a_{2-2}^2 = 2(0) + (-1)^2 = 1$
3	$a_3 = 3a_{3-1} + a_{3-2}^2 = 3(1) + (0)^2 = 3$
4	$a_4 = 4a_{4-1} + a_{4-2}^2 = 4(3) + (1)^2 = 13$
5	$a_5 = 5a_{5-1} + a_{5-2}^2 = 5(13) + (3)^2 = 74$

e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

n	$a_n = a_{n-1} - a_{n-2} + a_{n-3}$
0	$a_0 = 1$
1	$a_1 = 1$
2	$a_2 = 2$
3	$a_3 = a_{3-1} - a_{3-2} + a_{3-3} = 2 - 1 + 1 = 2$
4	$a_4 = a_{4-1} - a_{4-2} + a_{4-3} = 0 - 2 + 1 = 1$
5	$a_5 = a_{5-1} - a_{5-2} + a_{5-3} = 1 - 2 + 2 = 1$

11. Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$.

a) Find a_0, a_1, a_2, a_3, and a_4.

b) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

a) $a_n = 2^n + 5.3^n$

$a_0 = 2^0 + 5.3^0 = 6$

$a_1 = 2^1 + 5.3^1 = 17$

$a_2 = 2^2 + 5.3^2 = 49$

$a_3 = 2^3 + 5.3^3 = 143$

$a_4 = 2^4 + 5.3^4 = 421$

b)

$5a_1 - 6a_0 = 5(17) - 6(6) = 85 - 36 = 49 = a_2$

$5a_2 - 6a_1 = 5(49) - 6(17) = 245 - 102 = 143 = a_3$

$5a_3 - 6a_2 = 5(143) - 6(49) = 715 - 294 = 421 = a_4$

c) $a_n = 2^n + 5.3^n$

$5a_{n-1} - 6a_{n-2} =$

$5a_{n-1} = 5(2^{n-1} + 5.3^{n-1}) = 5.2^{n-1} + 25.3^{n-1}$

$6a_{n-2} = 6(2^{n-2} + 5.3^{n-2}) = 6.2^{n-2} + 30.3^{n-2}$

$5a_{n-1} - 6a_{n-2} = (5.2^{n-1} + 25.3^{n-1}) - (6.2^{n-2} + 30.3^{n-2})$

$= 5.2^{n-1} - 6.2^{n-2} + 25.3^{n-1} - 30.3^{n-2}$

$= 5.2.2^{n-2} - 6.2^{n-2} + 25.3.3^{n-2} - 30.3^{n-2}$

$= (10 - 6)2^{n-2} + (75 - 30)3^{n-2}$

$= 4.2^{n-2} + 45.3^{n-2}$

$= 2^n + 5.3^n$

$= a_n$

26. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, …
b) 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, …

e) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, …

a) $a = 3$

$a_n = n^2 + 2$

$A_{11} = 11^2 + 2 = 123$

$A_{12} = 12^2 + 2 = 146$

$A_{13} = 13^2 + 2 = 171$

next terms: 123, 146, 171

b) $a = 7$

$d = 4$

$a_n = 7 + 4(n - 1)$

$A_{11} = 7 + 4(11 - 1) = 47$

$A_{12} = 7 + 4(12 - 1) = 51$

$A_{13} = 7 + 4(13 - 1) = 55$

next terms: 47, 51, 55

e) $a=0$

$a_n = 3^n - 1$

$A_{11} = 3^{11} - 1 = 177146$

$A_{12} = 3^{12} - 1 = 531440$

$A_{13} = 3^{13} - 1 = 1594322$

next terms: 177146, 531440, 1594322

29. What are the values of these sums?

a) $\sum_{k=1}^{5}(k+1)$ **b)** $\sum_{j=0}^{4}(-2)^j$

c) $\sum_{i=1}^{10} 3$ **d)** $\sum_{j=0}^{8}(2^{j+1}-2^j)$

a) $\sum_{K=1}^{5}(K+1) = (1+1)+(2+1)+(3+1)+(4+1)+(5+1)$

$= 2+3+4+5+6 = 20$

b) $\sum_{j=0}^{4}(-2)^j = (-2)^0+(-2)^1+(-2)^2+(-2)^3+(-2)^4$

$= 1-2+4-8+16 = 11$

c) $\sum_{i=1}^{10} 3 = 3\sum_{i=1}^{10} 1 = 3.10 = 30$

d) $\sum_{j=0}^{8}(2^{j+1}-2^j) = (2^{0+1}-2^0)+(2^{1+1}-2^1)+(2^{2+1}-2^2)+(2^{3+1}-2^3)$

$+(2^{4+1}-2^4)+(2^{5+1}-2^5)+(2^{6+1}-2^6)+(2^{7+1}-2^7)+(2^{8+1}-2^8)$

$= 1+2+4+8+16+32+64+128+256 = 511$

30. What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

a) $\sum_{j \in S} j$ **b)** $\sum_{j \in S} j^2$

c) $\sum_{j \in S} (1/j)$ **d)** $\sum_{j \in S} 1$

a) $\sum_{j \in S} j = 1 + 3 + 5 + 7 = 16$

b) $\sum_{j \in S} j^2 = 1^2 + 3^2 + 5^2 + 7^2 = 1 + 9 + 25 + 49 = 84$

c) $\sum_{j \in S} (1/j) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

d) $\sum_{j \in S} 1 = 1 + 1 + 1 + 1 +$