IQ ACADEMY

Exercise set 0.4 :

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1, 3(a, b, c, d, f) 9, 10,11, 12, 13, 17,18, 19, 22 p.44-45.

1. In (a)–(d), determine whether f and g are inverse functions.

(a) $f(x) = 4x, \ g(x) = \frac{1}{4}x$

$(f \circ g)(X) = f(g(X)) = f(\frac{1}{4}X) = \cancel{4}(\frac{1}{\cancel{4}}X) = X$

$(g \circ f)(X) = g(f(x)) = g(4X) = \frac{1}{\cancel{4}}(\cancel{4}X) = X$

$\because$ f and g are inverse function

(b) $f(x) = 3x + 1, \ g(x) = 3x - 1$

$(f \circ g)(X) = f(g(X)) = f(3X - 1)$

$= 3(3X - 1) + 1 = 9X - 3 + 1$

$= 9X - 2$

$\because$ f and g are not inverse function

(c) $f(x) = \sqrt[3]{x-2},\ g(x) = x^3 + 2$

$(f\circ g)(x) = f(g(x)) = f(x^3+2) = \sqrt[3]{(x^3\cancel{+2})\cancel{-2}}$

$= \sqrt[3]{\cancel{x^3}} = x$

$(g\circ f)(x) = g(f(x)) = g(\sqrt[3]{x-2}) = (\cancel{\sqrt[3]{x-2}})^{\cancel{3}} + 2$

$= x \cancel{-2} \cancel{+2}$

$= x$

$\because$ f and g are inverse function

(d) $f(x) = x^4,\ g(x) = \sqrt[4]{x}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt[4]{x}) = (\cancel{\sqrt[4]{x}})^{\cancel{4}} = x$

$(g\circ f)(x) = g(f(x)) = g(x^4) = \cancel{\sqrt[4]{x^4}} = x$

$\because$ f and g are inverse function

3. In each part, use the horizontal line test to determine whether the function f is one-to-one.

(a) $f(x) = 3x + 2$

x	$y = 3x+2$
-2	$3(-2)+2 = -4$
-1	$3(-1)+2 = -1$
0	$3(0)+2 = 2$
1	$3(1)+2 = 5$
2	$3(2)+2 = 8$

(x, y)

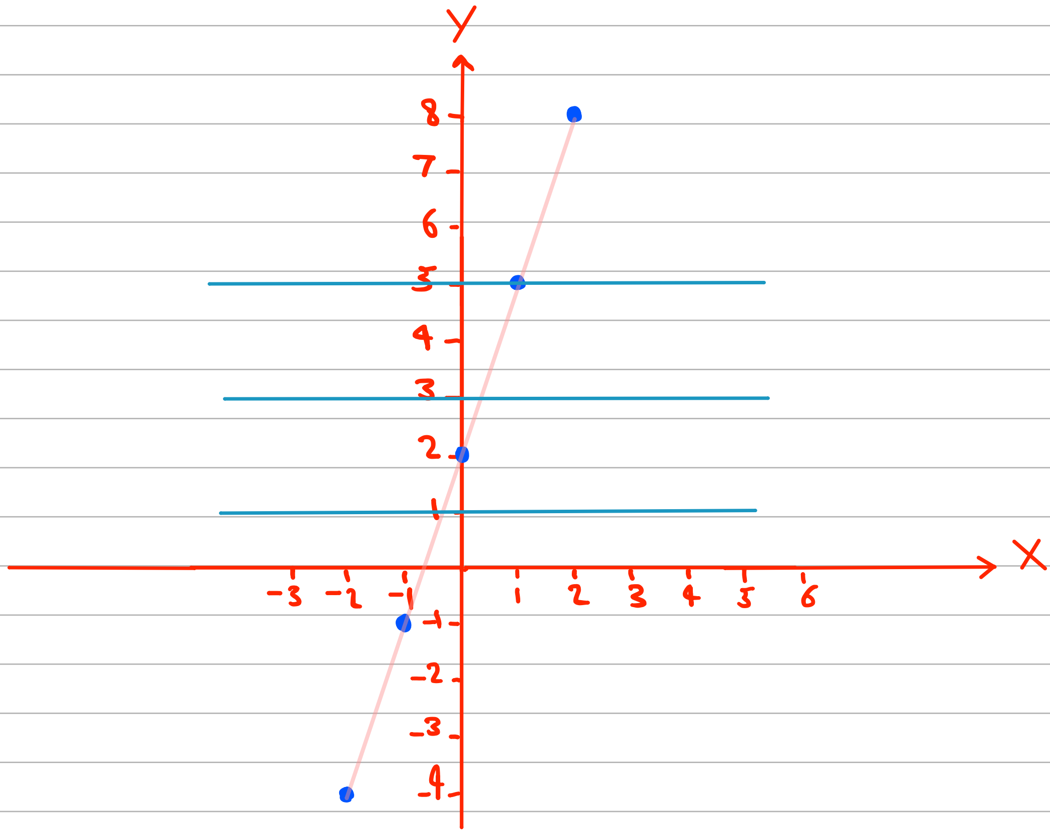
$(-2, -4)$

$(-1, -1)$

$(0, 2)$

$(1, 5)$

$(2, 8)$



one to one function

(b) $f(x) = \sqrt{x-1}$

X	$Y=\sqrt{X-1}$
1	$\sqrt{1-1}=0$
2	$\sqrt{2-1}=\sqrt{1}=1$
3	$\sqrt{3-1}=\sqrt{2}=1.4$
4	$\sqrt{4-1}=\sqrt{3}=1.7$

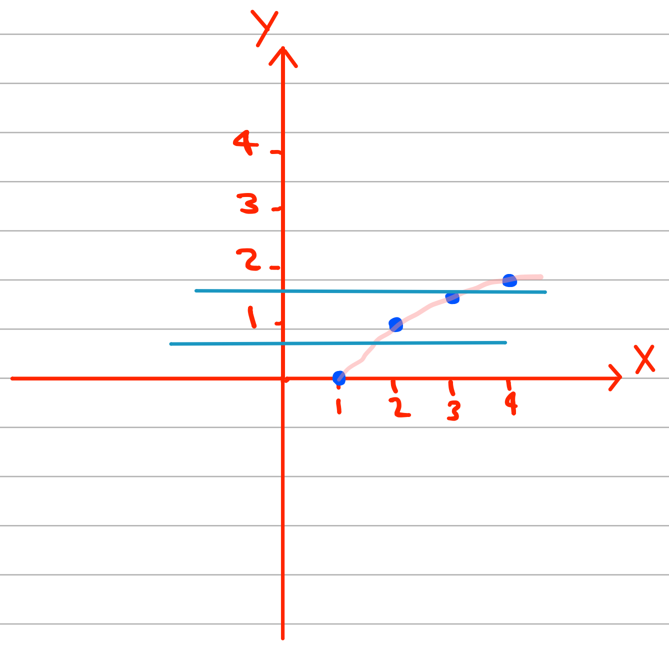
(X, Y)

$(1, 0)$

$(2, 1)$

$(3, 1.4)$

$(4, 1.7)$



one to one function

(c) $f(x) = |x|$

x	$y = f(x) = \lvert x \rvert$
-2	$\lvert -2 \rvert = 2$
-1	$\lvert -1 \rvert = 1$
0	$\lvert 0 \rvert = 0$
1	$\lvert 1 \rvert = 1$
2	$\lvert 2 \rvert = 2$

(x, y)

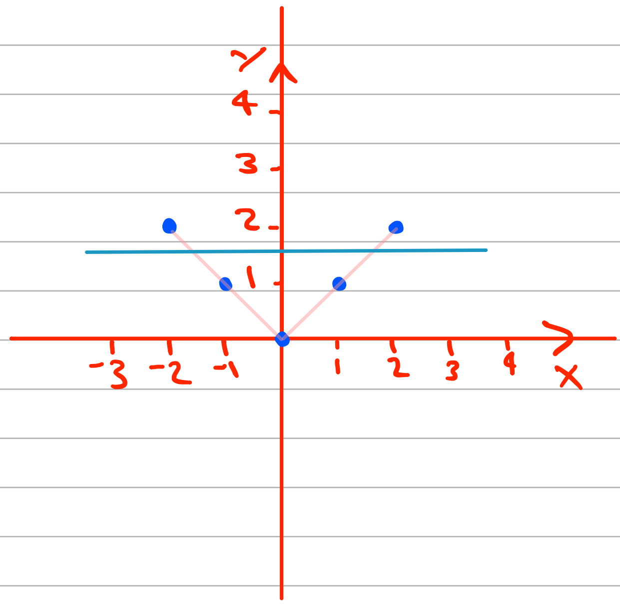
$(-2, 2)$

$(-1, 1)$

$(0, 0)$

$(1, 1)$

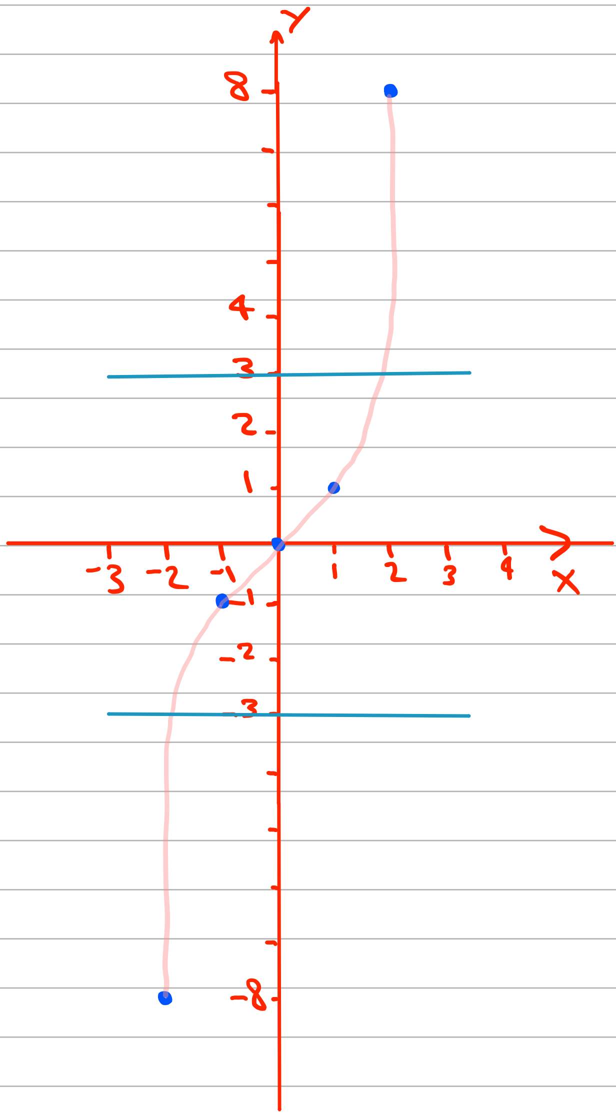
$(2, 2)$



not one to one function

(d) $f(x) = x^3$

x	$y=f(x)=x^3$	(x,y)
-2	$(-2)^3=-8$	$(-2,-8)$
-1	$(-1)^3=-1$	$(-1,-1)$
0	$(0)^3=0$	$(0,0)$
1	$(1)^3=1$	$(1,1)$
2	$(2)^3=8$	$(2,8)$



one to one function

(f) $f(x) = \sin x$

x	$y = \sin x$
$\frac{1}{2}\pi$	$\sin(\frac{1}{2}\pi) = 1$
π	$\sin(\pi) = 0$
2π	$\sin(2\pi) = 0$
$\frac{3}{2}\pi$	$\sin(\frac{3}{2}\pi) = -1$
$-\frac{3}{2}\pi$	$= 1$
-2π	$= 0$
$-\pi$	$= 0$
$-\frac{1}{2}\pi$	$= -1$

(x, y)

$(\frac{1}{2}\pi, 1)$

$(\pi, 0)$

$(2\pi, 0)$

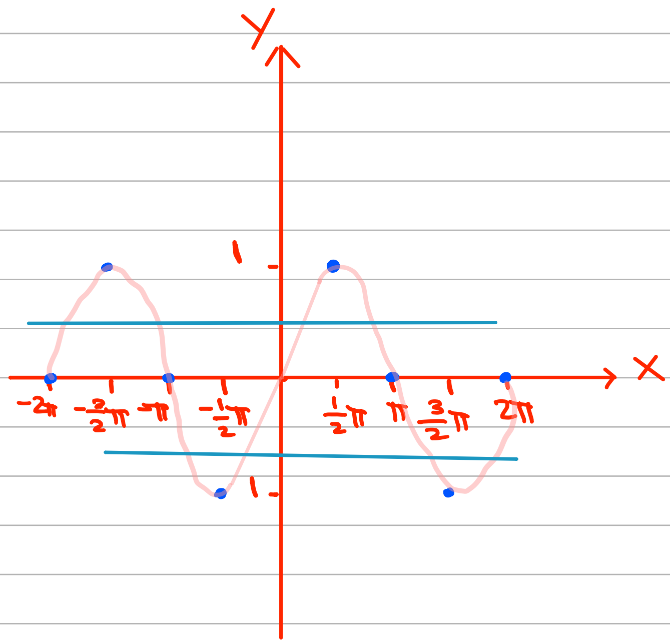
$(\frac{3}{2}\pi, -1)$

$(-\frac{3}{2}\pi, 1)$

$(-2\pi, 0)$

$(-\pi, 0)$

$(-\frac{1}{2}\pi, -1)$



one to one function

9–16 Find a formula for $f^{-1}(x)$.

9. $f(x) = 7x - 6$

$Y = 7X - 6$

$Y + 6 = 7X$

$X = \frac{Y+6}{7}$

$f^{-1}(y) = \frac{y+6}{7}$

$f^{-1}(x) = \frac{x+6}{7}$

10. $f(x) = \dfrac{x+1}{x-1}$

$$y = \frac{x+1}{x-1}$$

$$y(x-1) = (x+1)$$

$$yx - y = x+1$$

$$yx - x = y+1$$

$$x(y-1) = y+1$$

$$x = \frac{y+1}{y-1}$$

$$f^{-1}(y) = \frac{y+1}{y-1}$$

$$f^{-1}(x) = \frac{x+1}{x-1}$$

11. $f(x) = 3x^3 - 5$

$Y = 3X^3 - 5$

$Y + 5 = 3X^3$

$X^3 = \dfrac{Y+5}{3}$

$\sqrt[3]{X^3} = \sqrt[3]{\dfrac{Y+5}{3}}$

$X = \sqrt[3]{\dfrac{Y+5}{3}}$

$f^{-1}(y) = \sqrt[3]{\dfrac{Y+5}{3}}$

$f^{-1}(x) = \sqrt[3]{\dfrac{X+5}{3}}$

12. $f(x) = \sqrt[5]{4x+2}$

$Y = \sqrt[5]{4X+2}$

$(Y)^5 = (\sqrt[5]{4X+2})^5$

$Y^5 = 4X+2$

$Y^5 - 2 = 4X$

$X = \dfrac{Y^5 - 2}{4}$

$f^{-1}(y) = \dfrac{Y^5 - 2}{4}$

$f^{-1}(x) = \dfrac{X^5 - 2}{4}$

13. $f(x) = 3/x^2, \quad x < 0$

$$Y = \frac{3}{X^2}$$

$$YX^2 = 3$$

$$X^2 = \frac{3}{Y}$$

$$\sqrt{X^2} = \sqrt{\frac{3}{Y}}$$

$$X = \sqrt{\frac{3}{Y}}$$

$$f^{-1}(y) = \sqrt{\frac{3}{y}}$$

$$f^{-1}(x) = -\sqrt{\frac{3}{x}}$$

17–20 Find a formula for $f^{-1}(x)$, and state the domain of the function f^{-1}. ■

17. $f(x) = (x + 2)^4, \quad x \geq 0$

$$Y = (X+2)^4$$

$$\sqrt[4]{Y} = \sqrt[4]{(X+2)^4}$$

$$\sqrt[4]{Y} = X+2$$

$$X = \sqrt[4]{Y} - 2$$

$$f^{-1}(X) = \sqrt[4]{X} - 2$$

Domain:

$D_{f^{-1}} = [0, +\infty)$ because the Condition $X \geqslant 0$

18. $f(x) = \sqrt{x+3}$

$y = \sqrt{x+3}$

$(y)^2 = (\sqrt{x+3})^2$

$y^2 = x+3$

$x = y^2 - 3$

$f^{-1}(x) = x^2 - 3$

Domain:

$D_f = x+3 \geqslant 0 = x \geqslant -3 = [-3, +\infty)$

Range:

$x \geqslant -3 \longrightarrow x+3 \geqslant -3+3 \longrightarrow x+3 \geqslant 0$

$\sqrt{x+3} \geqslant \sqrt{0} \longrightarrow \sqrt{x+3} \geqslant 0 \longrightarrow [0, +\infty)$

$R_f = D_{f^{-1}} = [0, +\infty)$

$D_{f^{-1}} = [0, +\infty)$

19. $f(x) = -\sqrt{3-2x}$

$$y = -\sqrt{3-2x}$$

$$y^2 = \left(-\sqrt{3-2x}\right)^2$$

$$y^2 = 3-2x$$

$$y^2 - 3 = 2x$$

$$x = \frac{y^2-3}{2}$$

$$f^{-1}(x) = \frac{x^2-3}{2}$$

Domain:

$$D_f = 3-2x \geqslant 0 \rightarrow 3 \geqslant 2x \rightarrow x \leqslant \frac{3}{2} \rightarrow \left(-\infty, \frac{3}{2}\right]$$

Range:

$$x \leqslant \frac{3}{2} \rightarrow 2x \leqslant 3 \rightarrow 2x-3 \leqslant 3-3$$

$$\sqrt{2x-3} \leqslant \sqrt{0} \rightarrow \sqrt{2x-3} \leqslant 0 \rightarrow (-\infty, 0]$$

$$R_f = D_{f^{-1}} = (-\infty, 0]$$

$$D_{f^{-1}} = (-\infty, 0]$$

21–24 True–False Determine whether the statement is true or false. Explain your answer.

22. If f and g are inverse functions, then f and g have the same domain. (F)

$f(x) = 1 + \frac{1}{x} \longrightarrow D_f = R - \{0\}$

$g(x) = \frac{1}{x - 1} \longrightarrow D_g = R - \{1\}$