Exercise set (4.2):

**Exercise 4.2. P.214-215:**
**1-2(a)-6-9(c)-10(c)-11-16-25**

**EXERCISE SET 4.2** Graphing Utility C CAS

**1.** In each part, confirm that the formula is correct, and state a corresponding integration formula.

(a) $\frac{d}{dx}[\sqrt{1+x^2}] = \frac{x}{\sqrt{1+x^2}}$

(b) $\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2\cos(1+x^3)$

a) $\frac{d}{dx}(\sqrt{1+x^2}) = \frac{1}{2\sqrt{1+x^2}} \cdot 2x$

$= \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$

$\therefore$ formula is Correct

$\int \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2} + C$

b) $\frac{d}{dx}\left(\frac{1}{3}\sin(1+x^3)\right)$

$= \frac{1}{\cancel{3}} \cos(1+x^3) \cdot \cancel{3}x^2$

$= x^2 \cos(1+x^3)$

$\therefore$ formula is Correct

$$\int x^2 \cos(1+x^3)\,dx = \frac{1}{3}\sin(1+x^3) + C$$

**2.** In each part, confirm that the stated formula is correct by differentiating.

(a) $\displaystyle\int x \sin x \, dx = \sin x - x \cos x + C$

$$\frac{d}{dx}(\sin x - x\cos x + C) =$$

$$= \cos x - (\cos x + x(-\sin x))$$

$$= \cos x - (\cos x - x\sin x)$$

$$= \cancel{\cos x} - \cancel{\cos x} + x\sin x$$

$$= x\sin x$$

$\therefore$ Correct

**5–8** Find the derivative and state a corresponding integration formula.

**6.** $\dfrac{d}{dx}\left[\dfrac{x}{x^2+3}\right]$

$$= \frac{(x^2+3)(1) - x(2x)}{(x^2+3)^2}$$

$$= \frac{(x^2+3) - 2x^2}{(x^2+3)^2}$$

$$= \frac{-x^2+3}{(x^2+3)^2}$$

$$= \frac{3-x^2}{(x^2+3)^2}$$

$$\int \frac{3-x^2}{(x^2+3)^2}\,dx$$

$$= \frac{x}{x^2+3} + C$$

**9–10** Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule (Formula 2 in Table 4.2.1).

**9.** (a) $\int x^8\,dx$ (b) $\int x^{5/7}\,dx$ (c) $\int x^3\sqrt{x}\,dx$

**10.** (a) $\int \sqrt[3]{x^2}\,dx$ (b) $\int \frac{1}{x^6}\,dx$ (c) $\int x^{-7/8}\,dx$

9) c) $\int x^3 \sqrt{x}\,dx$

$\int x^3 x^{1/2}\,dx = \int x^{3+\frac{1}{2}}\,dx$

$\int x^{7/2}\,dx = \dfrac{x^{7/2+1}}{\frac{7}{2}+1} + C$

$= \dfrac{x^{9/2}}{9/2} + C = \dfrac{2}{9} x^{9/2} + C$

10) c) $\int x^{-7/8}\,dx$

$\dfrac{x^{-7/8+1}}{-\frac{7}{8}+1} + C$

$= \dfrac{x^{1/8}}{1/8} + C$

$= 8x^{1/8} + C$

$= 8\sqrt[8]{x} + C$

**11–14** Evaluate each integral by applying Theorem 4.2.3 and Formula 2 in Table 4.2.1 appropriately.

**11.** $\int \left[5x + \frac{2}{3x^5}\right] dx$

$\int 5x\,dx + \int \frac{2}{3x^5}\,dx$

$5\int x\,dx + \frac{2}{3}\int \frac{1}{x^5}\,dx$

$5\int x\,dx + \frac{2}{3}\int x^{-5}\,dx$

$5\ \frac{x^2}{2} + \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + C$

$= \frac{5x^2}{2} - \frac{2x^{-4}}{12} + C$

$= \frac{5x^2}{2} - \frac{x^{-4}}{6} + C$

$= \frac{5x^2}{2} - \frac{1}{6x^4} + C$

**15–30** Evaluate the integral and check your answer by differentiating.

**16.** $\int (2 + y^2)^2 \, dy$

$(a+b)^2 = a^2 + 2ab + b^2$

$\int (4 + 4y^2 + y^4) \, dy$

$\int 4 \, dy + 4 \int y^2 \, dy + \int y^4 \, dy$

$4y + 4 \frac{y^3}{3} + \frac{y^5}{5} + C$

_ Check the answer by differentiating

$* \frac{d}{dy} \left( 4y + \frac{4}{3} y^3 + \frac{1}{5} y^5 \right)$

$= 4 + 3\left(\frac{4}{3}\right) y^{3-1} + 5\left(\frac{1}{5}\right) y^4$

$= 4 + \frac{12}{3} y^2 + \frac{5}{5} y^4$

$= 4 + 4y^2 + y^4$

$= (2 + y^2)^2$

**25.** $\displaystyle\int \frac{\sec\theta}{\cos\theta}\,d\theta$

$\int \sec\theta \cdot \frac{1}{\cos\theta}\,d\theta$ $\boxed{\frac{1}{\cos\theta} = \sec\theta}$

$\int \sec\theta \cdot \sec\theta \, d\theta$

$\int \sec^2\theta \, d\theta$

$= \tan\theta + C$

- Check the answer by differentiating

$\frac{d}{d\theta}(\tan\theta + C) = \sec^2\theta$

$= \sec\theta \cdot \sec\theta$

$= \sec\theta \cdot \frac{1}{\cos\theta}$

$= \frac{\sec\theta}{\cos\theta}$