



## Exercise set (4.9):

Exercise set 4.9:

1(a,b,c), 2(a,b,c), 3, 4, 5, 6, 9, 10,  
11, 21, 22, 23, 24, 28, 29, 33, 34,  
37. p. 270 - 271

**1–4** Express the integral in terms of the variable  $u$ , but do not evaluate it. ■

1. (a)  $\int_1^3 (2x - 1)^3 dx$ ;  $u = 2x - 1$

$$du = 2 dx \rightarrow \frac{du}{2} = dx$$

$$\text{at } x = 1 \rightarrow u = 2(1) - 1 = 1$$

$$\text{at } x = 3 \rightarrow u = 2(3) - 1 = 5$$

$$= \frac{1}{2} \int_1^5 u^3 du$$

$$= \frac{1}{2} \cdot \left( \frac{u^4}{4} \right) \Big|_1^5$$

$$(b) \int_0^4 3x\sqrt{25-x^2} dx; u = 25 - x^2$$

$$u = 25 - x^2$$

$$du = -2x dx \rightarrow -\frac{du}{2} = x dx$$

$$\text{at } x = 0 \rightarrow u = 25 - 0^2 = 25$$

$$\text{at } x = 4 \rightarrow u = 25 - 4^2 = 9$$

$$\int_9^{25} 3\sqrt{u} \left(-\frac{du}{2}\right)$$

$$= -\frac{3}{2} \int_9^{25} 3\sqrt{u} du$$

$$(c) \int_{-1/2}^{1/2} \cos(\pi\theta) d\theta; u = \pi\theta$$

$$u = \pi\theta$$

$$du = \pi d\theta \rightarrow \frac{du}{\pi} = d\theta$$

$$\text{at } \theta = -\frac{1}{2} \rightarrow u = -\frac{\pi}{2}$$

$$\text{at } \theta = \frac{1}{2} \rightarrow u = \frac{\pi}{2}$$

$$\int_{-\pi/2}^{\pi/2} \cos(u) \frac{du}{\pi}$$

$$\frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(u) du$$

2. (a)  $\int_{-1}^4 (5 - 2x)^8 dx$ ;  $u = 5 - 2x$

$$du = -2 dx \rightarrow -\frac{du}{2} = dx$$

$$\text{at } x = -1 \rightarrow u = 5 - 2(-1) = 7$$

$$\text{at } x = 4 \rightarrow u = 5 - 2(4) = -3$$

$$= -\frac{1}{2} \int_7^{-3} u^8 du$$

$$= \frac{1}{2} \int_{-3}^7 u^8 du$$

(b)  $\int_{-\pi/3}^{2\pi/3} \frac{\sin x}{\sqrt{2 + \cos x}} dx$ ;  $u = 2 + \cos x$

$$du = -\sin x dx$$

$$\text{at } x = \frac{2\pi}{3} \rightarrow u = 2 + \cos\left(\frac{2\pi}{3}\right) = 2 + \left(-\frac{1}{2}\right) = \frac{3}{2}$$

$$\text{at } x = -\frac{\pi}{3} \rightarrow u = 2 + \cos\left(-\frac{\pi}{3}\right) = 2 + \frac{1}{2} = \frac{5}{2}$$

$$\int_{3/2}^{5/2} \frac{1}{\sqrt{u}} du$$

$$(c) \int_0^{\pi/4} \tan^2 x \sec^2 x dx; u = \tan x$$

$$du = \sec^2 x dx$$

$$\text{at } x=0 \rightarrow u = \tan 0 = 0$$

$$\text{at } x = \frac{\pi}{4} \rightarrow u = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\int_0^1 u^2 du$$

**3-12** Evaluate the definite integral two ways: first by a  $u$ -substitution in the definite integral and then by a  $u$ -substitution in the corresponding indefinite integral. ■

$$3. \int_0^1 (2x+1)^3 dx$$

$$u = 2x + 1$$

$$du = 2dx \rightarrow \frac{du}{2} = dx$$

$$\text{at } x=0 \rightarrow u = 2 \cdot 0 + 1 = 1$$

$$\text{at } x=1 \rightarrow u = 2 \cdot 1 + 1 = 3$$

$$\frac{1}{2} \int_1^3 u^3 du$$

$$= \frac{1}{2} \cdot \left( \frac{u^4}{4} \right)_1^3$$

$$= \frac{1}{8} (u^4)_1^3$$

$$= \frac{1}{8} (3^4 - 1^4)$$

$$= \frac{1}{8} (81 - 1) = 10$$

second method:

$$\frac{1}{2} \int_0^1 u^3 du$$

$$= \frac{1}{2} \cdot \left( \frac{u^4}{4} \right)_0^1$$

$$= \frac{1}{8} (u^4)'_0$$

$$= \frac{1}{8} ((2x+1)^4)'_0$$

$$= \frac{1}{8} ((2 \cdot 1 + 1)^4 - (2 \cdot 0 + 1)^4)$$

$$= \frac{1}{8} ((3)^4 - (1))$$

$$= \frac{1}{8} (81 - 1) = 10$$

$$4. \int_1^2 (4x - 2)^3 dx$$

$$u = 4x - 2$$

$$du = 4dx \rightarrow \frac{du}{4} = dx$$

$$\text{at } x=1 \rightarrow u = 4 \cdot 1 - 2 = 2$$

$$\text{at } x=2 \rightarrow u = 4 \cdot 2 - 2 = 6$$

$$\frac{1}{4} \int_2^6 u^3 du$$

$$= \frac{1}{4} \cdot \left( \frac{u^4}{4} \right)_2^6$$

$$= \frac{1}{16} (u^4)_2^6$$

$$= \frac{1}{16} (6^4 - 2^4)$$

$$= \frac{1}{16} (1296 - 16) = 80$$

second method:

$$\frac{1}{4} \int_1^2 u^3 du$$

$$= \frac{1}{4} \cdot \left( \frac{u^4}{4} \right)_1^2$$

$$= \frac{1}{16} (u^4)_1^2$$

$$= \frac{1}{16} (4x - 2)^4 \Big|_1^2$$

$$= \frac{1}{16} \left( (4 \cdot 2 - 2)^4 - (4 \cdot 1 - 2)^4 \right)$$

$$= \frac{1}{16} (1296 - 16) = 80$$

$$5. \int_0^1 (2x-1)^3 dx$$

$$u = 2x - 1$$

$$du = 2dx \rightarrow \frac{du}{2} = dx$$

$$\text{at } x=0 \rightarrow u = 2 \cdot 0 - 1 = -1$$

$$\text{at } x=1 \rightarrow u = 2 \cdot 1 - 1 = 1$$

$$\frac{1}{2} \int_{-1}^1 u^3 du$$

$$= \frac{1}{2} \cdot \left( \frac{u^4}{4} \right)_{-1}^1$$

$$= \frac{1}{8} (u^4)_{-1}^1$$

$$= \frac{1}{8} (1^4 - (-1)^4)$$

$$= \frac{1}{8} (1 - 1) = 0$$

second method:

$$\frac{1}{2} \int_0^1 u^3 du$$

$$= \frac{1}{2} \cdot \left( \frac{u^4}{4} \right)_0^1$$

$$= \frac{1}{8} \left( (2x-1)^4 - (2x-1)^4 \right) \Big|_0$$

$$= \frac{1}{8} \left( (2.1-1)^4 - (2.0-1)^4 \right)$$

$$= \frac{1}{8} (1-1) = 0$$

$$6. \int_1^2 (4-3x)^8 dx$$

$$u = 4 - 3x$$

$$du = -3 dx \rightarrow -\frac{du}{3} = dx$$

$$\text{at } x=1 \rightarrow u = 4 - 3 \cdot 1 = 1$$

$$\text{at } x=2 \rightarrow u = 4 - 3 \cdot 2 = -2$$

$$-\frac{1}{3} \int_1^{-2} u^8 du = \frac{1}{3} \int_{-2}^1 u^8 du$$

$$= \frac{1}{3} \cdot \left( \frac{u^9}{9} \right)_{-2}^1$$

$$= \frac{1}{27} (u^9)_{-2}^1$$

$$= \frac{1}{27} (1^9 - (-2)^9)$$

$$= \frac{1}{27} (1 - (-512)) = 19$$

second method:

$$-\frac{1}{3} \int_1^2 u^8 du$$

$$= -\frac{1}{3} \cdot \left( \frac{u^9}{9} \right)_1^2$$

$$= -\frac{1}{27} (\omega^9)_1^2$$

$$= -\frac{1}{27} \left( (4-3x)^9 \right)_1^2$$

$$= -\frac{1}{27} \left( (4-3.2)^9 - (4-3.1)^9 \right)$$

$$= -\frac{1}{27} \left( (-2)^9 - (1)^9 \right)$$

$$= -\frac{1}{27} (-512 - 1) = 19$$

$$9. \int_0^{\pi/2} 4 \sin(x/2) dx$$

$$u = x/2$$

$$du = \frac{1}{2} dx \rightarrow 2 du = dx$$

$$\text{at } x=0 \rightarrow u = 0/2 = 0$$

$$\text{at } x = \frac{\pi}{2} \rightarrow u = \frac{\pi/2}{2} = \frac{\pi}{4}$$

$$8 \int_0^{\pi/4} \sin(u) du$$

$$= 8(-\cos(u)) \Big|_0^{\pi/4}$$

$$= 8(-\cos(\frac{\pi}{4}) - (-\cos(0)))$$

$$= 8(-\frac{\sqrt{2}}{2} + 1)$$

$$= -4\sqrt{2} + 8$$

$$10. \int_0^{\pi/6} 2 \cos 3x \, dx$$

$$u = 3x$$

$$du = 3 \, dx \rightarrow \frac{du}{3} = dx$$

$$\text{at } x=0 \rightarrow u=0$$

$$\text{at } x = \frac{\pi}{6} \rightarrow u = \frac{3\pi}{6}$$

$$\frac{2}{3} \int_0^{3\pi/6} \cos(u) \, du$$

$$= \frac{2}{3} (\sin u)_0^{3\pi/6}$$

$$= \frac{2}{3} (\sin(\frac{3\pi}{6}) - \sin(0))$$

$$= \frac{2}{3} (\sin(\frac{\pi}{2}) - 0)$$

$$= \frac{2}{3} (1) = \frac{2}{3}$$

- second method:

$$\frac{2}{3} \int_0^{\pi/6} \cos(u) \, du$$

$$\frac{2}{3} (\sin(u))_0^{\pi/6}$$

$$= \frac{2}{3} \left( \sin(3x) \right) \Big|_0^{\pi/6}$$

$$= \frac{2}{3} \left( \sin\left(\frac{3\pi}{6}\right) - \sin(0) \right)$$

$$= \frac{2}{3} \left( \sin\left(\frac{\pi}{2}\right) - 0 \right)$$

$$= \frac{2}{3} (1) = \frac{2}{3}$$

$$11. \int_{-2}^{-1} \frac{x}{(x^2 + 2)^3} dx$$

$$u = x^2 + 2$$

$$du = 2x dx \rightarrow \frac{du}{2} = x dx$$

$$\text{at } x = -2 \rightarrow u = -2^2 + 2 = 6$$

$$\text{at } x = -1 \rightarrow u = -1^2 + 2 = 3$$

$$\frac{1}{2} \int_6^3 \frac{1}{u^3} du$$

$$= -\frac{1}{2} \int_6^3 \frac{1}{u^3} du$$

$$= -\frac{1}{2} \left( -\frac{1}{2u^2} \right)_6^3$$

$$= \frac{1}{2} \left( \frac{1}{2u^2} \right)_6^3$$

$$= \frac{1}{2} \left( \frac{1}{2 \cdot 6^2} - \frac{1}{2 \cdot 3^2} \right)$$

$$= \frac{1}{2} \cdot \frac{1}{24} = \frac{1}{48}$$

Second method:

$$\frac{1}{2} \int_{-2}^{-1} \frac{1}{u^3} du$$

$$= \frac{1}{2} \left( -\frac{1}{2u^2} \right)_{-2}^{-1}$$

$$= -\frac{1}{2} \left( \frac{1}{2u^2} \right)_{-2}^{-1}$$

$$= -\frac{1}{2} \left( \frac{1}{2(x^2+2)^2} \right)_{-2}^{-1}$$

$$= -\frac{1}{4} \left( \frac{1}{(-1^2+2)^2} - \frac{1}{(-2^2+2)^2} \right)$$

$$= -\frac{1}{4} \left( \frac{1}{9} - \frac{1}{36} \right)$$

$$= -\frac{1}{48}$$

**21–34** Evaluate the integrals by any method. ■

$$21. \int_1^5 \frac{dx}{\sqrt{2x-1}}$$

$$u = 2x - 1$$

$$du = 2dx \rightarrow \frac{du}{2} = dx$$

$$\frac{1}{2} \int \frac{1}{\sqrt{u}} = \frac{1}{2} \cdot 2\sqrt{u}$$

$$= \sqrt{u}$$

$$= (\sqrt{2x-1}) \Big|_1^5$$

$$= (\sqrt{2(5)-1} - \sqrt{2(1)-1})$$

$$= \sqrt{9} - \sqrt{1} = 3 - 1 = 2$$

$$22. \int_1^2 \sqrt{5x-1} dx$$

$$u = 5x - 1$$

$$du = 5 dx \rightarrow \frac{du}{5} = dx$$

$$\frac{1}{5} \int \sqrt{u} du$$

$$= \frac{1}{5} \cdot \frac{u^{1/2}}{1/2}$$

$$= \frac{2}{5} \cdot u^{1/2}$$

$$= \frac{2}{5} \left( (5x-1)^{1/2} \right)_1^2$$

$$= \frac{2}{5} \left( (5(2)-1)^{1/2} - (5(1)-1)^{1/2} \right)$$

$$= \frac{2}{5} \left( 9^{1/2} - 4^{1/2} \right) = \frac{38}{5}$$

$$23. \int_{-1}^1 \frac{x^2 dx}{\sqrt{x^3 + 9}}$$

$$u = x^3 + 9$$

$$du = 3x^2 dx \rightarrow \frac{du}{3} = x^2 dx$$

$$\frac{1}{3} \int \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} = \frac{2}{3} u^{1/2}$$

$$= \frac{2}{3} \left( (x^3 + 9)^{1/2} \right)_{-1}^1$$

$$= \frac{2}{3} \left( (1^3 + 9)^{1/2} - (-1^3 + 9)^{1/2} \right)$$

$$= \frac{2\sqrt{10} - 4\sqrt{2}}{3}$$

$$24. \int_{\pi/2}^{\pi} 6 \sin x (\cos x + 1)^5 dx$$

$$6 \int \sin x (\cos x + 1)^5 dx$$

$$u = \cos(x+1)$$

$$du = -\sin x dx \rightarrow -du = \sin x dx$$

$$-6 \int u^5 du$$

$$= -6 \frac{u^6}{6}$$

$$= -u^6$$

$$= -(\cos(x+1))^6 \Big|_{\pi/2}^{\pi}$$

$$= -\left( (\cos(\pi+1))^6 - (\cos(\frac{\pi}{2}+1))^6 \right)$$

$$= 1$$

$$28. \int_0^{\pi/4} \sqrt{\tan x} \sec^2 x \, dx$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\text{at } x=0 \quad \underline{\quad}, u = \tan(0) = 0$$

$$\text{at } x = \pi/4 \quad \underline{\quad}, u = \tan(\pi/4) = 1$$

$$\int_0^1 \sqrt{u} \, du = \int_0^1 (u)^{1/2} \, du$$

$$= \left( \frac{u^{3/2}}{3/2} \right)_0^1 = \frac{2}{3} \left( u^{3/2} \right)_0^1$$

$$= \frac{2}{3} \left( (1)^{3/2} - (0)^{3/2} \right)$$

$$= \frac{2}{3}$$

$$29. \int_0^{\sqrt{\pi}} 5x \cos(x^2) dx$$

$$u = x^2$$

$$du = 2x dx \rightarrow \frac{du}{2} = x dx$$

$$\text{at } x=0 \rightarrow u=0$$

$$\text{at } x=\sqrt{\pi} \rightarrow u = (\sqrt{\pi})^2 = \pi$$

$$\frac{5}{2} \int_0^{\pi} \cos(u) du$$

$$= \frac{5}{2} (\sin u)_0^{\pi}$$

$$= \frac{5}{2} (\sin \pi - \sin 0) = \frac{5}{2} (0 - 0) = 0$$

$$33. \int_0^1 \frac{y^2 dy}{\sqrt{4-3y}}$$

$$u = 4 - 3y$$

$$du = -3dy \rightarrow -\frac{du}{3} = dy$$

$$u = 4 - 3y \rightarrow y = -\frac{u-4}{3}$$

$$\text{at } y=0 \rightarrow u = 4 - 3(0) = 4$$

$$\text{at } y=1 \rightarrow u = 4 - 3(1) = 1$$

$$-\frac{1}{3} \int_4^1 \frac{\left(-\frac{u-4}{3}\right)^2}{\sqrt{u}} du$$

$$-\frac{1}{3} \int_4^1 \frac{(u-4)^2}{9\sqrt{u}} du = -\frac{1}{3} \int_4^1 \frac{(u-4)^2}{9\sqrt{u}} du$$

$$= -\frac{1}{27} \int_4^1 \frac{(u-4)^2}{(u)^{1/2}} du = -\frac{1}{27} \int_4^1 \frac{u^2 - 8u + 16}{(u)^{1/2}} du$$

$$= \frac{1}{27} \int_1^4 u^{3/2} - 8u^{1/2} + 16u^{-1/2} du$$

$$= \frac{1}{27} \left( \frac{u^{5/2}}{5/2} - 8 \frac{u^{3/2}}{3/2} - 16 \frac{u^{1/2}}{1/2} \right) \Big|_1^4$$

$$= \frac{1}{27} \left( \frac{2}{5} u^{5/2} - 8 \cdot \frac{2}{3} u^{3/2} - 16 \cdot 2 u^{1/2} \right)^4$$

$$= \frac{1}{27} \left( \frac{2}{5} u^{5/2} - \frac{16}{3} u^{3/2} - 32 u^{1/2} \right)^4$$

$$= \frac{1}{27} \left( \left( \frac{2}{5} (4)^{5/2} - \frac{16}{3} (4)^{3/2} - 32 (4)^{1/2} \right) \right.$$

$$\left. - \left( \frac{2}{5} (1)^{5/2} - \frac{16}{3} (1)^{3/2} - 32 (1)^{1/2} \right) \right)$$

$$= \frac{1}{27} \left( \left( \frac{64}{5} - \frac{128}{3} - 64 \right) - \left( -\frac{554}{15} \right) \right)$$

$$= \frac{1}{27} \left( \left( -\frac{1408}{15} \right) - \left( -\frac{554}{15} \right) \right)$$

$$34. \int_{-1}^4 \frac{x dx}{\sqrt{5+x}}$$

$$u = 5 + x$$

$$du = dx$$

$$x = u - 5$$

$$\text{at } x = -1 \rightarrow u = 5 + (-1) = 4$$

$$\text{at } x = 4 \rightarrow u = 5 + 4 = 9$$

$$\int_4^9 \frac{u-5}{\sqrt{u}} du = \int_4^9 \frac{u}{u^{1/2}} - \frac{5}{u^{1/2}} du$$

$$= \int_4^9 u^{1/2} - 5u^{-1/2} du$$

$$= \left( \frac{u^{3/2}}{3/2} - 5 \frac{u^{1/2}}{1/2} \right) \Big|_4^9 = \left( \frac{2}{3} u^{3/2} - 10u^{1/2} \right) \Big|_4^9 =$$

$$= \left( \frac{2}{3} (9^{3/2}) - 10(9^{1/2}) \right) - \left( \frac{2}{3} (4^{3/2}) - 10(4^{1/2}) \right)$$

$$= 18 - 30 - \frac{16}{3} + 20 = \frac{8}{3}$$

37. (a) Find  $\int_0^1 f(3x+1) dx$  if  $\int_1^4 f(x) dx = 5$ .

(b) Find  $\int_0^3 f(3x) dx$  if  $\int_0^9 f(x) dx = 5$ .

(c) Find  $\int_{-2}^0 xf(x^2) dx$  if  $\int_0^4 f(x) dx = 1$ .

a)  $w = 3x + 1$

$$dw = 3 dx \rightarrow \frac{dw}{3} = dx$$

at  $x = 0 \rightarrow w = 3 \cdot 0 + 1 = 1$

at  $x = 1 \rightarrow w = 3 \cdot 1 + 1 = 4$

$$\int_1^4 f(w) \frac{dw}{3} = \frac{1}{3} \int_1^4 f(w) dw$$

$$= \frac{1}{3} \cdot 5 = \frac{5}{3}$$

b)  $w = 3x$

$$dw = 3 dx \rightarrow \frac{dw}{3} = dx$$

at  $x = 0 \rightarrow w = 3 \cdot 0 = 0$

at  $x = 3 \rightarrow w = 3 \cdot 3 = 9$

$$\int_0^9 f(w) \frac{dw}{3} = \frac{1}{3} \int_0^9 f(w) dw$$

$$= \frac{1}{3} \cdot 5 = \frac{5}{3}$$

$$c) u = 2x$$

$$du = 2x dx \rightarrow \frac{du}{2} = x dx$$

$$\text{at } x = -2 \rightarrow u = -2^2 = 4$$

$$\text{at } x = 0 \rightarrow u = 0^2 = 0$$

$$\frac{1}{2} \int_4^0 f(u) du = -\frac{1}{2} \int_0^4 f(u) du$$

$$= -\frac{1}{2} (1)$$

$$= -\frac{1}{2}$$