Exercise set (4.6):

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5,6,7,8,9(a),10,11,12,13,14,
15,16,17,18,19,20,47(a),49,
50, 53(a,b), 54(a,b). p. 251 - 252

5–8 Find the area under the curve $y = f(x)$ over the stated interval.

5. $f(x) = x^3$; $[2, 3]$

$\int_2^3 x^3 \, dx$

$= \frac{x^4}{4} \Big]_2^3$

$= \frac{3^4}{4} - \frac{2^4}{4}$

$= \frac{81}{4} - \frac{16}{4}$

$= \frac{65}{4}$

6. $f(x) = x^4;\ [-1, 1]$

$$\int_{-1}^{1} x^4\,dx$$

$$= \frac{x^5}{5}\Big]_{-1}^{1}$$

$$= \frac{1^5}{5} - \frac{-1^5}{5}$$

$$= \frac{1^5}{5} + \frac{1^5}{5}$$

$$= \frac{2}{5}$$

7. $f(x) = 3\sqrt{x};\ [1, 4]$

$\int_1^4 3\sqrt{x}\,dx$

نستبعد الثابت لخارج التكامل

$= 3\int_1^4 x^{1/2}\,dx$

نكامل

$= 3 \cdot \frac{x^{3/2}}{3/2} \Big]_1^4$

ناخذ المقام لانه كسر الى فوق ولكن يكون الكسر مقلوب البسط مقام والمقام بسط

$= \cancel{3} \cdot \frac{2}{\cancel{3}} x^{3/2} \Big]_1^4$

نختصر

$= 2x^{3/2} \Big]_1^4$

الان نعوض في حدود التكامل

$= 2\left[4^{3/2} - 1^{3/2}\right]$

$= 2(8 - 1)$

$= 14$

8. $f(x) = x^{-2/3};\ [1, 27]$

$\int_1^{27} x^{-2/3}\, dx$ نكامل

$= \left.\frac{x^{1/3}}{1/3}\right]_1^{27}$ ناخذ المقام لانه كسر الى فوق ولكن يكون الكسر مقلوب البسط مقام والمقام بسط

$= \left. 3x^{1/3}\right]_1^{27}$ الان نعوض في حدود التكامل

$= 3\left[27^{1/3} - 1^{1/3}\right]$

$= 3(3 - 1)$

$= 6$

9–10 Find all values of x^* in the stated interval that satisfy Equation (8) in the Mean-Value Theorem for Integrals (4.6.2), and explain what these numbers represent. ■

9. (a) $f(x) = \sqrt{x}$; $[0, 3]$

$\int_0^3 \sqrt{x}\, dx = f(x^*)(b-a)$

$= \frac{x^{3/2}}{3/2}\Big]_0^3 = \frac{2}{3} x^{3/2}\Big]_0^3$

$= \frac{2}{3}\left(3^{3/2} - 0^{3/2}\right)$

$= \frac{2}{\cancel{3}}\left(\cancel{3}\sqrt{3}\right)$

$= 2\sqrt{3}$

$2\sqrt{3} = \sqrt{x^*}\,(3-0)$

$\frac{2\sqrt{3}}{3} = \sqrt{x^*}$

$\left(\frac{2\sqrt{3}}{3}\right)^2 = \left(\sqrt{x^*}\right)^2$

$x^* = \frac{4}{3}$

9–10 Find all values of x^* in the stated interval that satisfy Equation (8) in the Mean-Value Theorem for Integrals (4.6.2), and explain what these numbers represent.

10. (a) $f(x) = \sin x;\ [-\pi, \pi]$

$\int_{-\pi}^{\pi} \sin x\,dx = f(x^*)(b-a)$

$= -\cos x \Big]_{-\pi}^{\pi}$

$= -(\cos(\pi) - \cos(-\pi))$

$= -(1-(-1)) = 0$

$0 = f(x^*)(b-a)$

$0 = \sin x^*(\pi-(-\pi))$

$0 = \sin x^*(2\pi)$

$0 = 2\pi \sin x^*$

$\frac{0}{2\pi} = \frac{\cancel{2\pi} \sin x^*}{\cancel{2\pi}}$

$\sin x^* = 0$

$\cancel{\sin^{-1}}(\cancel{\sin} x^*) = \sin^{-1}(0)$

$x^* = \sin^{-1}(0)$

$x^* = \pm\pi$

(b) $f(x) = 1/x^2;\ [1, 3]$

$\int_1^3 \frac{1}{x^2}\,dx$

نرفع ال x تربيع من المقام للبسط فيكون في البسط واشارته سالبة

$= \int_1^3 x^{-2}\,dx$

نكامل

$= \frac{x^{-1}}{-1}\Big]_1^3 = -x^{-1}\Big]_1^3$

نعوض في حدود التكامل

$= \left(-\frac{1}{3}\right) - \left(-\frac{1}{1}\right) = -\frac{1}{3} + 1 = \frac{2}{3}$

اوجدنا ناتج الطرف الايسر من القانون

$\int_1^3 \frac{1}{x^2}\,dx = f(x^*)(b - a)$

$\frac{2}{3} = \frac{1}{(x^*)^2}(3 - 1)$

$\frac{2}{3} = \frac{2}{(x^*)^2}$

$2(x^*)^2 = 6$

$\frac{\cancel{2}(x^*)^2}{\cancel{2}} = \frac{6}{2}$

$(x^*)^2 = 3$

$\sqrt{(x^*)^{\cancel{2}}} = \sqrt{3}$

$x^* = \pm\sqrt{3}$

$x^* = \sqrt{3} \in [1, 3]$

$x^* = -\sqrt{3} \notin [1, 3]$

11–22 Evaluate the integrals using Part 1 of the Fundamental Theorem of Calculus.

11. $\int_{-2}^{1} (x^2 - 6x + 12)\, dx$

$$\int_{-2}^{1} (x^2 - 6x + 12)\, dx$$

$$= \int_{-2}^{1} x^2\, dx - \int_{-2}^{1} 6x\, dx + \int_{-2}^{1} 12\, dx$$

$$= \frac{x^3}{3} - \frac{6x^2}{2} + 12x \Big]_{-2}^{1}$$

$$= \frac{x^3}{3} - 3x^2 + 12x \Big]_{-2}^{1}$$

$$= \left(\frac{1^3}{3} - 3(1)^2 + 12(1)\right) - \left(\frac{-2^3}{3} - 3(-2)^2 + 12(-2)\right)$$

$$= \frac{1}{3} - 3 + 12 - \left(\frac{-8}{3} - 3 \cdot 4 - 24\right)$$

$$= \frac{1}{3} + 9 - \left(\frac{-8}{3} - 12 - 24\right)$$

$$= \frac{1}{3} + 9 - \left(\frac{-8}{3} - 36\right)$$

$$= \frac{1}{3} + 9 + \frac{116}{3}$$

$$= 48$$

12. $\int_{-1}^{2} 4x(1 - x^2)\, dx$

$\int_{-1}^{2} 4x - x^3\, dx$

$= \int_{-1}^{2} 4x\, dx - \int_{-1}^{2} x^3\, dx$

$= \frac{4x^2}{2} - \frac{4x^4}{4} \Big]_{-1}^{2}$

$= 2x^2 - x^4 \Big]_{-1}^{2}$

$= (2(2)^2 - 2^4) - (2(-1)^2 - (-1)^4)$

$= (8 - 16) - (2 - 1) = -8 - 1 = -9$

13. $\int_1^4 \frac{4}{x^2}\,dx$

$4\int_1^4 \frac{1}{x^2}\,dx$

$= 4\left(-\frac{1}{x}\right)\Big]_1^4$

$= -\frac{4}{x}\Big]_1^4$

$= -\frac{4}{4} - \left(-\frac{4}{1}\right)$

$= -1 + \frac{4}{1}$

$= 3$

14. $\int_1^2 \frac{1}{x^6}\,dx$

$$\int_1^2 \frac{1}{x^6}\,dx$$

$$= \left(-\frac{1}{5x^5}\right)\Big]_1^2$$

$$= -\frac{1}{5 \cdot 2^5} - \left(-\frac{1}{5 \cdot 1^5}\right)$$

$$= -\frac{1}{5 \cdot 32} + \frac{1}{5}$$

$$= \frac{31}{160}$$

15. $\int_4^9 2x\sqrt{x}\,dx$

$\int_4^9 2x \cdot x^{1/2}\,dx$

$= 2\int_4^9 x^{3/2}\,dx$

$= 2\left.\frac{x^{5/2}}{5/2}\right]_4^9$

$= \left.\frac{4}{5}x^{5/2}\right]_4^9$

$= \frac{4}{5}\left(9^{5/2} - 4^{5/2}\right)$

$= \frac{4}{5}\left(243 - 32\right)$

$= \frac{844}{5}$

16. $\displaystyle\int_1^4 \frac{1}{x\sqrt{x}}\,dx$

$$\int_1^4 x^{-1}x^{-1/2}\,dx = \int_1^4 x^{-3/2}\,dx$$

$$= \frac{x^{-1/2}}{-1/2}\Big]_1^4 = -2x^{-1/2}\Big]_1^4$$

$$= -\frac{2}{\sqrt{x}}\Big]_1^4 = -\left(\left(\frac{2}{\sqrt{4}}\right) - \left(\frac{2}{\sqrt{1}}\right)\right)$$

$$= -\left(\frac{2}{2} - \frac{2}{1}\right) = -(1-2) = -(-1) = 1$$

17. $\displaystyle\int_{-\pi/2}^{\pi/2} \sin\theta\,d\theta$

$$= -\cos\theta\Big]_{-\pi/2}^{\pi/2}$$

$$= -\cos\left(\frac{\pi}{2}\right) - \left(-\cos\left(-\frac{\pi}{2}\right)\right)$$

$$= 0 + 0$$

$$= 0$$

18. $\int_0^{\pi/4} \sec^2\theta \, d\theta$

$= \tan\theta \Big]_0^{\pi/4}$

$= \left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right)$

$= 1 - 0 \quad = 1$

19. $\int_{-\pi/4}^{\pi/4} \cos x \, dx$

$= \sin x \Big]_{-\pi/4}^{\pi/4}$

$= \left(\sin\left(-\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\right)$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$

$= \sqrt{2}$

20. $\int_0^{\pi/3} (2x - \sec x \tan x)\, dx$

$= 2\int_0^{\pi/3} x\, dx - \int_0^{\pi/3} \sec x \tan x\, dx$

$= \cancel{2}\, \frac{x^2}{\cancel{2}} \Big]_0^{\pi/3} - \sec x \Big]_0^{\pi/3}$

$= x^2 \Big]_0^{\pi/3} - \sec x \Big]_0^{\pi/3}$

$= \left(\left(\frac{\pi}{3}\right)^2 - (0)^2 \right) - \left(\sec\left(\frac{\pi}{3}\right) - \sec(0) \right)$

$= \frac{\pi^2}{9} - (2 - 1)$

$= \frac{\pi^2}{9} - 1$

47. Define $F(x)$ by

$$F(x) = \int_1^x (3t^2 - 3)\, dt$$

(a) Use Part 2 of the Fundamental Theorem of Calculus to find $F'(x)$.

$$F'(x) = \cancel{\frac{d}{dx}} \cancel{\int_1^x} (3t^2 - 3)\, dt$$

$$F'(x) = 3x^2 - 3$$

49–52 Use Part 2 of the Fundamental Theorem of Calculus to find the derivatives.

49. (a) $\dfrac{d}{dx}\displaystyle\int_1^x \sin(t^2)\,dt$ (b) $\dfrac{d}{dx}\displaystyle\int_1^x \sqrt{1-\cos t}\,dt$

a) $\dfrac{d}{\cancel{dx}}\cancel{\displaystyle\int_1^x} \sin(t^2)\,\cancel{dt}$

$F'(x) = \sin(x^2)$

b) $\dfrac{d}{\cancel{dx}}\cancel{\displaystyle\int_1^x}\sqrt{1-\cos t}\ dt$

$F'(x) = \sqrt{1-\cos x}$

50. (a) $\dfrac{d}{dx}\displaystyle\int_0^x \frac{dt}{1+\sqrt{t}}$ (b) $\dfrac{d}{dx}\displaystyle\int_2^x \frac{dt}{t^2+3t-4}$

a)

$$F'(x) = \cancel{\frac{d}{dx}} \cancel{\int_0^x} \frac{dt}{1+\sqrt{t}}$$

$$F'(x) = \frac{1}{1+\sqrt{x}}$$

b)

$$F'(x) = \cancel{\frac{d}{dx}} \cancel{\int_2^x} \frac{1}{t^2+3t-4}\, dt$$

$$F'(x) = \frac{1}{x^2+3x-4}$$

53. Let $F(x) = \int_4^x \sqrt{t^2+9}\,dt$. Find

(a) $F(4)$ (b) $F'(4)$

a) $\int_4^4 \sqrt{t^2+9}\,dt = 0$

$= \int_4^4 (t^2)^{1/2} + 9^{1/2}\,dt$

$= \int_4^4 t + 9^{1/2}\,dt$

$= \frac{t^2}{2} + 9^{1/2}t \Big]_4^4$

$= \left(\frac{4^2}{2} + 9^{1/2}(4)\right) - \left(\frac{4^2}{2} + 9^{1/2}(4)\right)$

$= 0$

b) $F'(4)$

$\frac{d}{dx} \cancel{\int_4^x} \sqrt{t^2+9}\,dt$

$F'(x) = \sqrt{x^2+9}$

$F'(4) = \sqrt{4^2+9} = \sqrt{25}$

54. Let $F(x) = \int_0^x \frac{\cos t}{t^2 + 3t + 5}\, dt$. Find

(a) $F(0)$ (b) $F'(0)$ (c) $F''(0)$.

a) $F(0) = \int_0^x \frac{\cos t}{t^2+3t+5}\, dt$

$= 0$

b) $F'(x) = \cancel{\frac{d}{dx}} \cancel{\int_0^x} \frac{\cos t}{t^2+3t+5}\, dt$

$= \frac{\cos x}{x^2+3x+5}$

$F'(0) = \frac{\cos 0}{0^2+3.0+5} = \frac{1}{5}$