



Section (4.2) :

4.2 THE INDEFINITE INTEGRAL

■ ANTIDERIVATIVES معكوس الاشتقاق

4.2.1 DEFINITION A function F is called an *antiderivative* of a function f on a given open interval if $F'(x) = f(x)$ for all x in the interval.

الدالة F تسمى معكوس المشتقة للدالة f على الفترة المفتوحة اذا كان $F'(x) = f(x)$ لكل قيم x في الفترة

$$F(x) = \frac{x^3}{3}$$

$$f(x) = x^2$$

$$F'(x) = \frac{d}{dx} \left(\frac{x^3}{3} \right)$$

$$= 3 \cdot \frac{x^2}{3}$$

$$= x^2$$

$F(x)$ is antiderivative of $f(x)$

4.2.2 THEOREM If $F(x)$ is any antiderivative of $f(x)$ on an open interval, then for any constant C the function $F(x) + C$ is also an antiderivative on that interval. Moreover, each antiderivative of $f(x)$ on the interval can be expressed in the form $F(x) + C$ by choosing the constant C appropriately.

$$F(x) = \frac{1}{3}x^3 + 4$$

$$F'(x) = \frac{d}{dx} \left(\frac{1}{3}x^3 + 4 \right)$$

$$= \frac{\cancel{3}}{\cancel{3}} x^2 + 0$$

$$F'(x) = x^2$$

THE INDEFINITE INTEGRAL

The process of finding antiderivatives is called **antidifferentiation** or **integration**. Thus, if

$$\frac{d}{dx}[F(x)] = f(x) \quad (1)$$

then **integrating** (or **antidifferentiating**) the function $f(x)$ produces an antiderivative of the form $F(x) + C$. To emphasize this process, Equation (1) is recast using **integral notation**,

$$\int f(x) dx = F(x) + C \quad (2)$$

where C is understood to represent an arbitrary constant. It is important to note that (1) and (2) are just different notations to express the same fact. For example,

$$\int x^2 dx = \frac{1}{3}x^3 + C \quad \text{is equivalent to} \quad \frac{d}{dx} \left[\frac{1}{3}x^3 \right] = x^2$$

Note that if we differentiate an antiderivative of $f(x)$, we obtain $f(x)$ back again. Thus,

$$\frac{d}{dx} \left[\int f(x) dx \right] = f(x) \quad (3)$$

The expression $\int f(x) dx$ is called an **indefinite integral**. The adjective “indefinite” emphasizes that the result of antidifferentiation is a “generic” function, described only up to a constant term. The “elongated s” that appears on the left side of (2) is called an **integral sign**,* the function $f(x)$ is called the **integrand**, and the constant C is called the **constant of integration**. Equation (2) should be read as:

The integral of $f(x)$ with respect to x is equal to $F(x)$ plus a constant.

INTEGRATION FORMULAS

Table 4.2.1

INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA	DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\frac{d}{dx}[x] = 1$	$\int dx = x + C$	5. $\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
2. $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \, (r \neq -1)$	$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \, (r \neq -1)$	6. $\frac{d}{dx}[-\cot x] = \csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$
3. $\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x \, dx = \sin x + C$	7. $\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
4. $\frac{d}{dx}[-\cos x] = \sin x$	$\int \sin x \, dx = -\cos x + C$	8. $\frac{d}{dx}[-\csc x] = \csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$

► **Example 1** The second integration formula in Table 4.2.1 will be easier to remember if you express it in words:

To integrate a power of x (other than -1), add 1 to the exponent and divide by the new exponent.

Here are some examples:

$$\int x^2 dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

$$\int x^3 dx = \frac{x^{3+1}}{3+1} + C = \frac{x^4}{4} + C$$

$$\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-5+1}}{-5+1} + C$$

$$= \frac{x^{-4}}{-4} + C = -\frac{1}{4x^4} + C$$

$$\int \sqrt{x} dx = \int x^{1/2} dx = \frac{x^{1/2+1}}{1/2+1} + C$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + C = \frac{2}{3} x^{3/2} + C$$

$$= \frac{2}{3} (x^{1/2})^3 + C$$

$$= \frac{2}{3} (\sqrt{x})^3 + C$$

سميزات التكامل غير المحدود

■ PROPERTIES OF THE INDEFINITE INTEGRAL

Our first properties of antiderivatives follow directly from the simple constant factor, sum, and difference rules for derivatives.

4.2.3 THEOREM Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that c is a constant. Then:

(a) A **constant factor can be moved through an integral sign**; that is,

$$\int cf(x) dx = cF(x) + C$$

(b) An **antiderivative of a sum is the sum of the antiderivatives**; that is,

$$\int [f(x) + g(x)] dx = F(x) + G(x) + C$$

(c) An **antiderivative of a difference is the difference of the antiderivatives**; that is,

$$\int [f(x) - g(x)] dx = F(x) - G(x) + C$$

The statements in Theorem 4.2.3 can be summarized by the following formulas:

$$\int cf(x) dx = c \int f(x) dx \quad (4)$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx \quad (5)$$

$$\int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx \quad (6)$$

► **Example 2** Evaluate

(a) $\int 4 \cos x \, dx$ → نستبدل الثابتة لخارج التكامل

$= 4 \int \cos x \, dx$ → تكامل ال Cos وتكاملها يساوي Sin

$= 4 \sin x + C$ → نضيف الثابتة C

(b) $\int (x + x^2) \, dx$

$\int x \, dx + \int x^2 \, dx$ → نوزع التكامل

$= \frac{x^2}{2} + \frac{x^3}{3} + C$ → تكامل المتغير x بأن نزود على الأس واحد ونقسم على الأس الجديد

► Example 3

$$\int (3x^6 - 2x^2 + 7x + 1) dx = 3 \int x^6 dx - 2 \int x^2 dx + 7 \int x dx + \int 1 dx$$

$$\int 3x^6 dx - \int 2x^2 dx + \int 7x dx + \int 1 dx \longrightarrow \text{نوزع الثابت على جميع الحدود}$$

$$3 \int x^6 dx - 2 \int x^2 dx + 7 \int x dx + \int 1 dx \longrightarrow \text{نستبعد الثابت ل خارج الثابت}$$

$$3 \frac{x^{6+1}}{6+1} - 2 \frac{x^{2+1}}{2+1} + 7 \frac{x^{1+1}}{1+1} + x + C \longrightarrow \text{ثابت المتغير بزيادة واحد على الاس ونقسم على الاس الجديد}$$

$$3 \frac{x^7}{7} - 2 \frac{x^3}{3} + 7 \frac{x^2}{2} + x + C$$

► **Example 4** Evaluate

(a) $\int \frac{\cos x}{\sin^2 x} dx$

$\int \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} dx \longrightarrow$ قمنا بتوزيع المقام على البسط بفك الـ $\sin$ تربيع لـ حاصل ضرب

$= \int \csc x \cot x dx \longrightarrow \csc x = \frac{1}{\sin x}, \cot x = \frac{\cos x}{\sin x}$

$= -\csc x + C$

(b) $\int \frac{t^2 - 2t^4}{t^4} dt$

$\int \frac{t^2}{t^4} - \frac{2t^4}{t^4} dt \longrightarrow$ وزعنا المقام على البسط

$\int \frac{1}{t^2} dt - \int 2 dt$

$\int t^{-2} dt - \int 2 dt \longrightarrow$ نرفع t تربيع للبسط فيتحول الأس لـ سالب

$= \frac{t^{-2+1}}{-2+1} - 2t + C$

$= \frac{t^{-1}}{-1} - 2t + C$

$= -\frac{1}{t} - 2t + C$