Exercise set (4.3):

Exercise 4.3. P.221-222:

1-2(a-b)-4(b)-5-6(b)-7-15-20-21-27-29-33

1–12 Evaluate the integrals using the indicated substitutions.

1. (a) $\int 2x(x^2+1)^{23}\,dx;\ u = x^2+1$

(b) $\int \cos^3 x \sin x\,dx;\ u = \cos x$

a) $u = x^2 + 1$

$du = 2x\,dx$

$\int u^{23}\,du$

$= \dfrac{u^{23+1}}{23+1} + C$

$= \dfrac{u^{24}}{24} + C$

$= \dfrac{(x^2+1)^{24}}{24} + C$

b) $u = \cos x$

$du = -\sin x \rightarrow -du = \sin x$

$\int (u^3) \cdot -du = -\int u^3\,du$

$= -\dfrac{u^{3+1}}{3+1} + C$

$= -\dfrac{u^4}{4} + C$

$= -\dfrac{\cos^4 x}{4} + C$

2. (a) $\displaystyle\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx;\ u = \sqrt{x}$

$u = \sqrt{x}$

$du = \dfrac{1}{2\sqrt{x}} dx$

$2du = \dfrac{1}{\sqrt{x}} dx$

$\int \sin(u) . 2du$

$= 2\int \sin(u) du$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

(b) $\int \frac{3x\,dx}{\sqrt{4x^2+5}}$; $u = 4x^2 + 5$

$u = 4x^2 + 5$

$du = 8x\,dx \longrightarrow \frac{du}{8} = x\,dx$

$= 3\int \frac{du}{8\sqrt{u}} = \frac{3}{8}\int \frac{du}{\sqrt{u}}$

$= \frac{3}{8} \cdot 2\sqrt{u} + C$

$= \frac{6}{8}\sqrt{4x^2+5} + C$

$= \frac{3}{4}\sqrt{4x^2+5} + C$

3. (a) $\int \sec^2(4x+1)\,dx;\ u = 4x+1$

$du = 4\,dx \longrightarrow \frac{du}{4} = dx$

$= \int \sec^2(u)\,\frac{du}{4} = \frac{1}{4}\int \sec^2(u)\,du$

$= \frac{1}{4}\tan u + C$

$= \frac{1}{4}\tan(4x+1) + C$

(b) $\int y\sqrt{1+2y^2}\,dy;\ u = 1+2y^2$

$du = 4y\,dy \longrightarrow \frac{du}{4} = y\,dy$

$= \frac{1}{4}\int \sqrt{u}\,du$

$= \frac{1}{4}\int (u)^{1/2}\,du$

$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{4}\cdot\frac{2}{3}\,u^{3/2} + C$

$= \frac{2}{12}\,u^{3/2} + C$

$= \frac{1}{6}(1+2y^2)^{3/2} + C$

4. (b) $\int (2x+7)(x^2+7x+3)^{4/5}\,dx;\; u = x^2+7x+3$

$u = x^2 + 7x + 3$

$du = 2x + 7$

$\int u^{4/5}\,du$

$= \dfrac{u^{4/5+1}}{4/5+1} + C$

$= \dfrac{u^{9/5}}{9/5} + C$

$= \dfrac{5}{9} u^{9/5} + C$

$= \dfrac{5}{9} (x^2 + 7x + 3)^{9/5} + C$

5. (a) $\int \cot x \csc^2 x \, dx; \quad u = \cot x$

(b) $\int (1 + \sin t)^9 \cos t \, dt; \quad u = 1 + \sin t$

a) $u = \cot x$

$du = -\csc^2 x \, dx \longrightarrow -du = \csc^2 x \, dx$

$\int u(-du) = -\int u \, du$

$= -\frac{u^2}{2} + C$

$= -\frac{\cot^2 x}{2} + C$

b) $u = 1 + \sin t$

$du = \cos t \, dt$

$\int u^9 \, du$

$= \frac{u^{10}}{10} + C$

$= \frac{(1 + \sin t)^{10}}{10} + C$

6. (a) $\int \cos 2x \, dx;\ u = 2x$

$du = 2dx \rightarrow \frac{du}{2} = dx$

$= \frac{1}{2} \int \cos(u)\, du$

$= \frac{1}{2} \sin(u) + C$

$= \frac{1}{2} \sin(2x) + C$

(b) $\int x \sec^2 x^2 \, dx; \; u = x^2$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \sec^2 u \cdot \frac{du}{2}$

$= \frac{1}{2} \int \sec^2 u \, du$

$= \frac{1}{2} (\tan u) + C$

$= \frac{1}{2} \tan x^2 + C$

7. (a) $\int x^2\sqrt{1+x}\,dx$; $u = 1 + x$

(b) $\int [\csc(\sin x)]^2 \cos x\,dx$; $u = \sin x$

a) $u = 1 + x$

$du = dx$

$u = 1 + x \longrightarrow u - 1 = x \longrightarrow (u-1)^2 = x^2$

$\int (u-1)^2 \sqrt{u}\,du$

$= \int (u^2 - 2u + 1)(u^{1/2})\,du = \int (u^{2+1/2} - 2u^{1+1/2} + u^{1/2})\,du$

$= \int (u^{5/2} - 2u^{3/2} + u^{1/2})\,du$

$= \frac{u^{5/2+1}}{5/2+1} - 2\frac{u^{3/2+1}}{3/2+1} + \frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{u^{7/2}}{7/2} - 2\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C$

$= \frac{2}{7}u^{7/2} - 2\left(\frac{2}{5}\right)u^{5/2} + \frac{2}{3}u^{3/2} + C$

$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$= \frac{2}{7}(1+x)^{7/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C$

b) $u = \sin x$

$du = \cos x \, dx$

$\int (\csc(u))^2 \, du$

$= -\cot u + C$

$= -\cot(\sin x) + C$

11–36 Evaluate the integrals using appropriate substitutions.

11. $\int (4x-3)^9\,dx$

$u = 4X - 3$

$du = 4\,dx \longrightarrow \frac{du}{4} = dx$

$\frac{1}{4}\int u^9\,du$

$= \frac{1}{4} \cdot \frac{u^{10}}{10} + C$

$= \frac{1}{40}u^{10} + C$

$= \frac{1}{40}(4X-3)^{10} + C$

12. $\int x^3\sqrt{5+x^4}\,dx$

$u = 5 + x^4$

$du = 4x^3\,dx \longrightarrow \frac{du}{4} = x^3\,dx$

$\frac{1}{4}\int \sqrt{u}\,du$

$= \frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + C$

$= \frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{2}{12} u^{3/2} + C$

$= \frac{1}{6} (5 + x^4)^{3/2} + C$

13. $\int \sin 7x\, dx$

$u = 7x$

$du = 7\, dx \longrightarrow \frac{du}{7} = dx$

$\frac{1}{7}\int \sin(u)\, du$

$= -\frac{1}{7}\cos(u) + C$

$= -\frac{1}{7}\cos(7x) + C$

14. $\int \cos \frac{x}{3}\, dx$

$u = x/3$

$du = \frac{1}{3}\, dx \longrightarrow 3\, du = dx$

$= 3\int \cos(u)\, du$

$= 3\sin(u) + C$

$= 3\sin\left(\frac{x}{3}\right) + C$

15. $\int \sec 4x \tan 4x \, dx$

$u = 4x$

$du = 4\,dx \longrightarrow \frac{du}{4} = dx$

$\int \sec(u)\tan(u)\frac{du}{4}$

$= \frac{1}{4}\int \sec(u)\tan(u)\,du$

$= \frac{1}{4}\sec(u) + C$

$= \frac{1}{4}\sec(4x) + C$

16. $\int \sec^2 5x \, dx$

$u = 5X$

$du = 5\,dX \longrightarrow \frac{du}{5} = dX$

$\frac{1}{5} \int \sec^2(u)\, du$

$= \frac{1}{5} \tan(u) + C$

$= \frac{1}{5} \tan(5X) + C$

17. $\int t\sqrt{7t^2+12}\,dt$

$w = 7t^2 + 12$

$dw = 14t\,dt \longrightarrow \frac{dw}{14} = t\,dt$

$\frac{1}{14}\int \sqrt{w}\;dw$

$= \frac{1}{14} \cdot \frac{w^{3/2}}{3/2} + C$

$= \frac{1}{14} \cdot \frac{2}{3} w^{3/2} + C$

$= \frac{2}{42} w^{3/2} + C$

$= \frac{1}{21} w^{3/2} + C$

$= \frac{1}{21}(7t^2+12)^{3/2} + C$

18. $\displaystyle\int \frac{x}{\sqrt{4-5x^2}}\,dx$

$u = 4 - 5X^2$

$du = -10X\,dX \longrightarrow -\dfrac{du}{10} = X\,dX$

$-\dfrac{1}{10}\displaystyle\int \frac{du}{\sqrt{u}}$

$= -\dfrac{1}{10}\displaystyle\int u^{-1/2}\,du$

$= -\dfrac{1}{10} \cdot \dfrac{u^{1/2}}{1/2} + C$

$= -\dfrac{1}{10} \cdot 2\,u^{1/2} + C$

$= -\dfrac{1}{5}\,u^{1/2} + C$

$= -\dfrac{1}{5}(4 - 5X^2)^{1/2} + C$

$= -\dfrac{1}{5}\sqrt{4 - 5X^2} + C$

19. $\displaystyle\int \frac{6}{(1-2x)^3}\,dx$

$u = 1 - 2X$

$du = -2\,dX \longrightarrow -\dfrac{du}{2} = dX$

$-\dfrac{6}{2}\displaystyle\int \frac{1}{u^3}\,du$

$= -3\displaystyle\int u^{-3}\,du$

$= -3 \cdot \dfrac{u^{-2}}{-2} + C$

$= \dfrac{3}{2} \cdot \dfrac{1}{u^2} + C$

$= \dfrac{3}{2(1-2X)^2} + C$

20. $\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$

$u = x^3 + 3x$

$du = 3x^2 + 3\,dx \longrightarrow du = 3(x^2+1)\,dx$

$\frac{du}{3} = (x^2+1)\,dx$

$\frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int \frac{du}{u^{1/2}}$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\cdot\frac{u^{-1/2+1}}{-1/2+1} + C$

$= \frac{1}{3}\cdot\frac{u^{1/2}}{\frac{1}{2}} + C$

$= \frac{1}{3}\cdot\frac{2}{1}\,u^{1/2} + C$

$= \frac{2}{3}\sqrt{u} + C$

$= \frac{2}{3}\sqrt{x^3+3x} + C$

21. $\displaystyle\int \frac{x^3}{(5x^4+2)^3}\,dx$

$u = (5x^4+2)$

$du = 20x^3\,dx \longrightarrow \frac{du}{20} = x^3\,dx$

$\int \frac{1}{u^3} \cdot \frac{du}{20}$

$= \frac{1}{20} \int \frac{1}{u^3}\,du$

$= \frac{1}{20} \int u^{-3}\,du$

$= \frac{1}{20} \cdot \frac{u^{-2}}{-2} + C$

$= -\frac{1}{40} u^{-2} + C$

$= -\frac{1}{40} \cdot \frac{1}{u^2} + C$

$= -\frac{1}{40(5x^4+2)^2} + C$

22. $\displaystyle\int \frac{\sin(1/x)}{3x^2}\,dx$

$u = 1/X$

$du = -\dfrac{1}{X^2}\,dX \longrightarrow -du = \dfrac{1}{X^2}\,dX$

$-\dfrac{1}{3}\displaystyle\int \sin(u)\,du$

$= -\dfrac{1}{3}(-\cos u) + C$

$= \dfrac{1}{3}\cos(1/X) + C$

23. $\int \frac{\sin(5/x)}{x^2}\,dx$

$u = 5/x$

$du = -5/x^2\,dx \longrightarrow -\frac{du}{5} = \frac{1}{x^2}\,dx$

$-\frac{1}{5}\int \sin(u)\,du$

$= -\frac{1}{5}(-\cos u) + C$

$= \frac{1}{5}\cos\left(\frac{5}{x}\right) + C$

24. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx \longrightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$

$= 2\int \sec^2 u\,du$

$= 2\tan u + C$

$= 2\tan\sqrt{x} + C$

25. $\int \cos^4 3t \sin 3t \, dt$

$u = \cos 3t$

$du = -3\sin 3t \, dt \longrightarrow -\frac{du}{3} = \sin 3t \, dt$

$-\frac{1}{3}\int u^4 \, du$

$= -\frac{1}{3} \cdot \frac{u^5}{5} + C$

$= -\frac{u^5}{15} + C$

$= -\frac{(\cos 3t)^5}{15} + C$

26. $\int \cos 2t \sin^5 2t \, dt$

$u = \sin 2t$

$du = 2\cos(2t)dt \longrightarrow \frac{du}{2} = \cos(2t)$

$\frac{1}{2}\int u^5 du$

$= \frac{1}{2} \cdot \frac{u^6}{6} + C$

$= \frac{1}{12} u^6 + C$

$= \frac{\sin(2t)^6}{12} + C$

27. $\int x \sec^2(x^2)\,dx$

$u = x^2$

$du = 2x\,dx \longrightarrow \frac{du}{2} = x\,dx$

$\frac{1}{2}\int \sec^2(u)\,du$

$= \frac{1}{2}\tan(u) + C$

$= \frac{1}{2}\tan(x^2) + C$

29. $\int \cos 4\theta \sqrt{2 - \sin 4\theta}\, d\theta$

$u = 2 - \sin 4\theta$

$du = -\cos 4\theta \cdot 4\, d\theta$

$-\frac{du}{4} = \cos 4\theta\, d\theta$

$\int \sqrt{u} \cdot -\frac{du}{4}$

$-\frac{1}{4} \int u^{1/2}\, du$

$= -\frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + C$

$= -\frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C$

$= -\frac{2}{12} u^{3/2} + C$

$= -\frac{1}{6} (2 - \sin 4\theta)^{3/2} + C$

30. $\int \tan^3 5x \sec^2 5x \, dx$

$u = \tan(5x)$

$du = 5\sec^2(5x)\,dx \longrightarrow \frac{du}{5} = \sec^2(5x)\,dx$

$\frac{1}{5}\int u^3\,du$

$= \frac{1}{5} \cdot \frac{u^4}{4} + C$

$= \frac{1}{20} \cdot u^4 + C$

$= \frac{\tan^4(5x)}{20} + C$

31. $\int \sec^3 2x \tan 2x \, dx$

$\int \sec^2(2x)\sec(2x)\tan(2x)dx$

$u = \sec 2x$

$du = 2\sec 2x \tan(2x)dx$

$\frac{du}{2} = \sec 2x \tan(2x)dx$

$\frac{1}{2}\int u^2 du$

$= \frac{1}{2} \cdot \frac{u^3}{3} + C$

$= \frac{1}{6} u^3 + C$

$= \frac{(\sec 2x)^3}{6} + C$

33. $\int \frac{y}{\sqrt{2y+1}}\,dy$

$u = 2y + 1$

$du = 2\,dy \longrightarrow \frac{du}{2} = dy$

$u = 2y + 1 \longrightarrow u - 1 = 2y$

$y = \frac{u-1}{2}$

$$\int \frac{u-1}{2\sqrt{u}} \cdot \frac{du}{2}$$

$$= \frac{1}{4}\int \frac{u-1}{u^{1/2}}\,du$$

$$= \frac{1}{4}\int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du$$

$$= \frac{1}{4}\int u^{1/2} - u^{-1/2}\,du$$

$$= \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + C$$

$$= \frac{1}{4} \cdot \frac{2}{3}\,u^{3/2} - \frac{1}{4} \cdot 2\,u^{1/2} + C$$

$$= \frac{2}{12}\,u^{3/2} - \frac{2}{4}\,u^{1/2} + C$$

$$= \frac{1}{6}\,u^{3/2} - \frac{1}{2}\,u^{1/2} + C$$

$$= \frac{1}{6}(2Y+1)^{3/2} - \frac{1}{2}(2Y+1)^{1/2} + C$$

34. $\int x\sqrt{4-x}\,dx$

$u = 4 - x \longrightarrow x = 4 - u$

$-du = dx$

$-\int (4-u)\sqrt{u}\,du$

$= -\int (4-u)(u)^{1/2}\,du$

$= -\int (4u^{1/2} - u^{3/2})\,du$

$= -\left(\frac{4u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right) + C$

$= -\frac{8}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C$

$= -\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + C$