Exercise set (4.5):

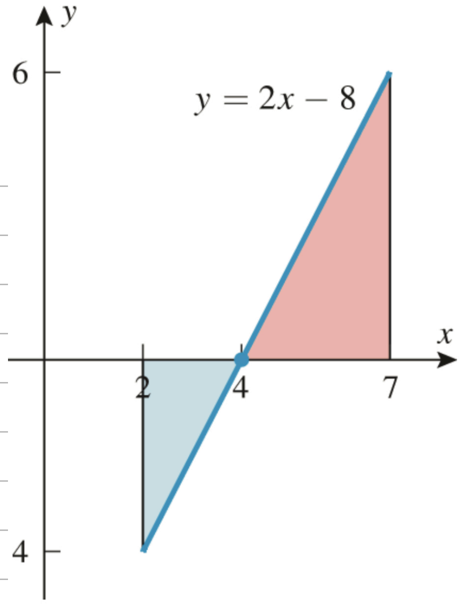
Exercise 4.5 P.239: **Quick check ex.3**

P. 240: **14-15(b)-16(a-b)-17(a-c-d)-22-23-24-33(b)**

✓ QUICK CHECK EXERCISES 4.5 (See page 242 for answers.)

3. Use the accompanying figure to evaluate

$$\int_2^7 (2x - 8)\,dx$$



◀ **Figure Ex-3**

$$A = A_1 + (-A_2) = A_1 - A_2$$

$$= \frac{1}{2} b_1 h_1 - \frac{1}{2} b_2 h_2$$

$$= \frac{1}{2}(6)(3) - \frac{1}{2}(4)(2)$$

$$= \frac{18}{2} - \frac{8}{2}$$

$$= 9 - 4 = 5$$

4. Suppose that $g(x)$ is a function for which

$$\int_{-2}^{1} g(x)\,dx = 5 \quad \text{and} \quad \int_{1}^{2} g(x)\,dx = -2$$

(a) $\int_{1}^{2} 5g(x)\,dx$

$= 5\int_1^2 g(x)\,dx$

$= 5(-2)$

$= -10$

(b) $\int_{-2}^{2} g(x)\,dx$

$\int_{-2}^{2} g(x)\,dx = \int_{-2}^{1} g(x)\,dx + \int_{1}^{2} g(x)\,dx$

$= 5 + (-2)$

$= 5 - 2$

$= 3$

(c) $\int_{1}^{1} [g(x)]^2\,dx$

$= 0$

(d) $\int_{2}^{-2} 4g(x)\,dx$

$-4\int_{-2}^{2} g(x)\,dx = -\int_{-2}^{1} g(x)\,dx - \int_{1}^{2} g(x)\,dx$

$= 4(-5-(-2))$

$= 4(-5+2) = -12$

13–16 Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed.

13. (a) $\int_0^3 x\,dx$

$Y = X$

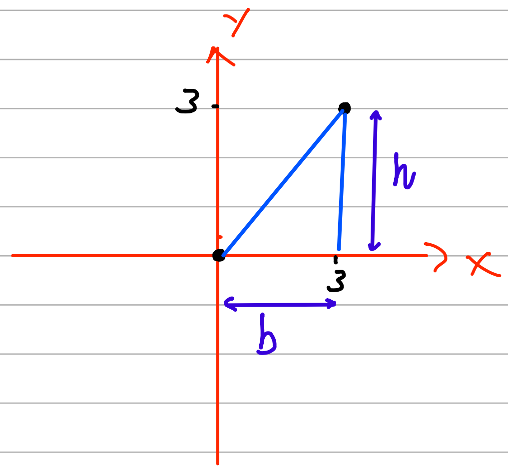
at $X = 0 \longrightarrow Y = 0$ $(0, 0)$

at $X = 3 \longrightarrow Y = 3$ $(3, 3)$

A = area of triangle

$= \frac{1}{2}(\text{base} \cdot \text{height})$

$= \frac{1}{2}(3)(3) = \frac{9}{2}$

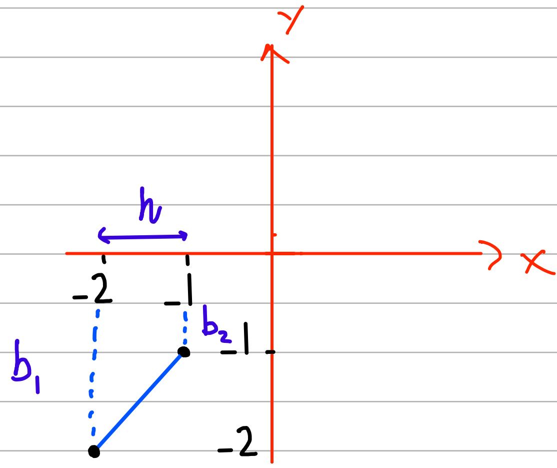


(b) $\int_{-2}^{-1} x \, dx$

$Y = X$

at $X = -2 \longrightarrow Y = -2$ $(-2, -2)$

at $X = -1 \longrightarrow Y = -1$ $(-1, -1)$



A = area of trapezoid

$= -\frac{1}{2}(\text{base}1 + \text{base}2) \cdot \text{height}$

$= -\frac{1}{2}(2 + 1) \cdot (1)$

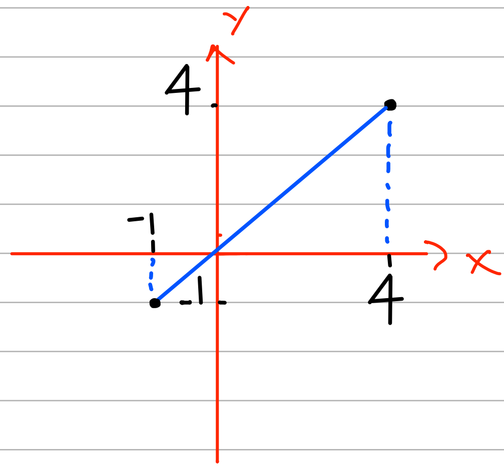
$= -\frac{3}{2}$

(c) $\int_{-1}^{4} x\,dx$

$Y = X$

at $X = -1 \longrightarrow Y = -1$ $(-1, -1)$

at $X = 4 \longrightarrow Y = 4$ $(4, 4)$



A = area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(4)(4) - \frac{1}{2}(1)(1)$

$= \frac{16}{2} - \frac{1}{2}$

$= 8 - \frac{1}{2}$

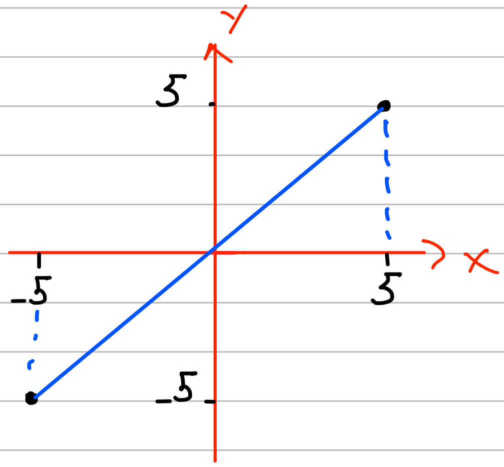
$= \frac{15}{2}$

(d) $\int_{-5}^{5} x\,dx$

$Y = X$

at $X = -5 \longrightarrow Y = -5$ $(-5, -5)$

at $X = 5 \longrightarrow Y = 5$ $(5, 5)$



$A =$ area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(5)(5) - \frac{1}{2}(5)(5)$

$= 0$

14. (a) $\int_0^2 \left(1 - \frac{1}{2}x\right)\, dx$ (b) $\int_{-1}^1 \left(1 - \frac{1}{2}x\right)\, dx$

(c) $\int_2^3 \left(1 - \frac{1}{2}x\right)\, dx$ (d) $\int_0^3 \left(1 - \frac{1}{2}x\right)\, dx$

a) $Y = 1 - \frac{1}{2}X$

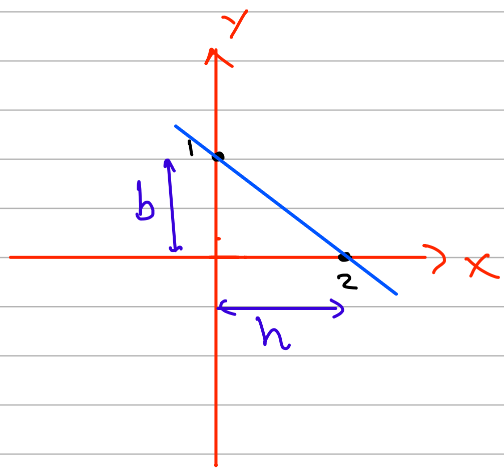
at $X = 0 \rightarrow Y = 1 - \frac{1}{2}(0) = 1$ $(0, 1)$

at $X = 2 \rightarrow Y = 1 - \frac{1}{2}(2) = 0$ $(2, 0)$

A = area of triangle

$= \frac{1}{2}(\text{base} \cdot \text{height})$

$= \frac{1}{2}(1)(2) = 1$



b) $\int_{-1}^{1} (1-\frac{1}{2}x)\,dx$

$Y = 1 - \frac{1}{2}X$

at $X = 1 \rightarrow Y = 1 - \frac{1}{2}(1) = \frac{1}{2}$ $\quad (1, \frac{1}{2})$

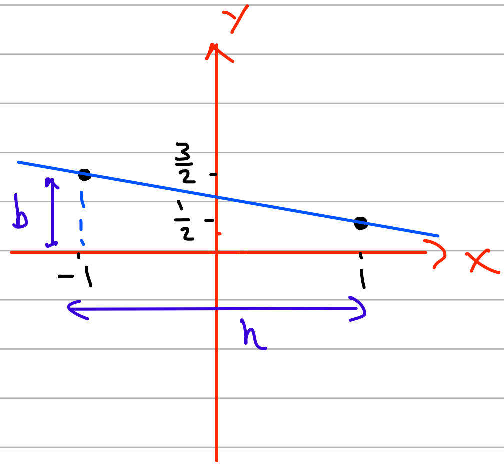
at $X = -1 \rightarrow Y = 1 - \frac{1}{2}(-1) = \frac{3}{2}$ $\quad (-1, \frac{3}{2})$

A = area of trapezoid

$= \frac{1}{2}(\text{base1} + \text{base2}).\text{height}$

$= \frac{1}{2}(\frac{3}{2} + \frac{1}{2}).2$

$= 2$

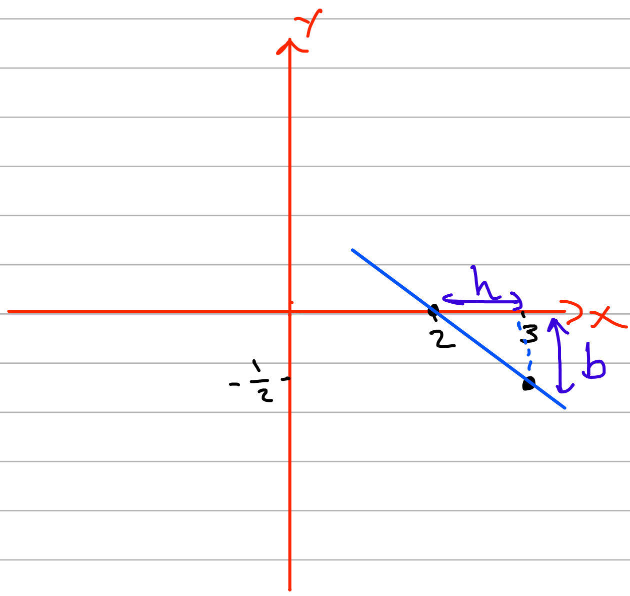


c) $\int_2^3 (1-\frac{1}{2}x)\,dx$

$Y = 1 - \frac{1}{2}X$

at $X = 2 \rightarrow Y = 1 - \frac{1}{2}(2) = 1 - 1 = 0$ $(2, 0)$

at $X = 3 \rightarrow Y = 1 - \frac{1}{2}(3) = -\frac{1}{2}$ $(3, -\frac{1}{2})$



A = area of triangle

$= -\frac{1}{2}(\text{base} \cdot \text{height})$

$= -\frac{1}{2}(\frac{1}{2})(1) = -\frac{1}{4}$

d) $\int_0^3 (1-\frac{1}{2}x)\,dx$

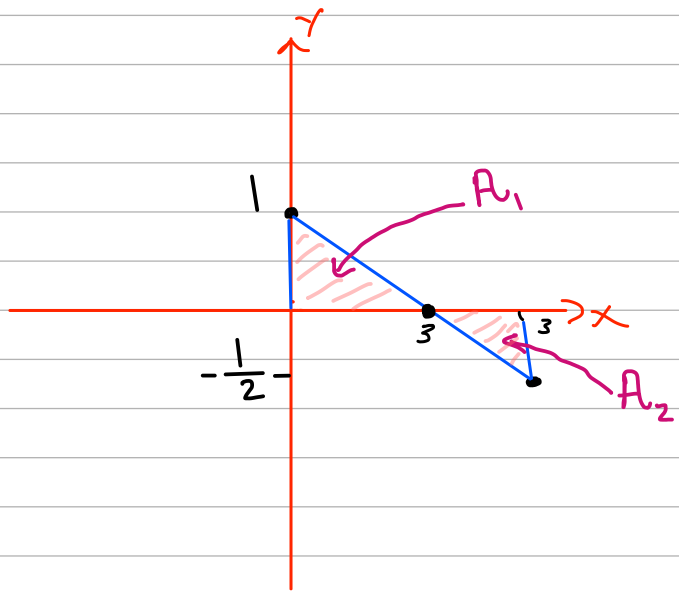
$Y = 1 - \frac{1}{2}X$

at $X=0 \rightarrow Y = 1 - \frac{1}{2}(0) = 1$ $\quad (0,1)$

at $X=3 \rightarrow Y = 1 - \frac{1}{2}(3) = -\frac{1}{2}$ $\quad (3,-\frac{1}{2})$

$Y=0 \rightarrow 0 = 1 - \frac{1}{2}X \rightarrow \frac{X}{2} = 1$

$X = 2$



A = area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

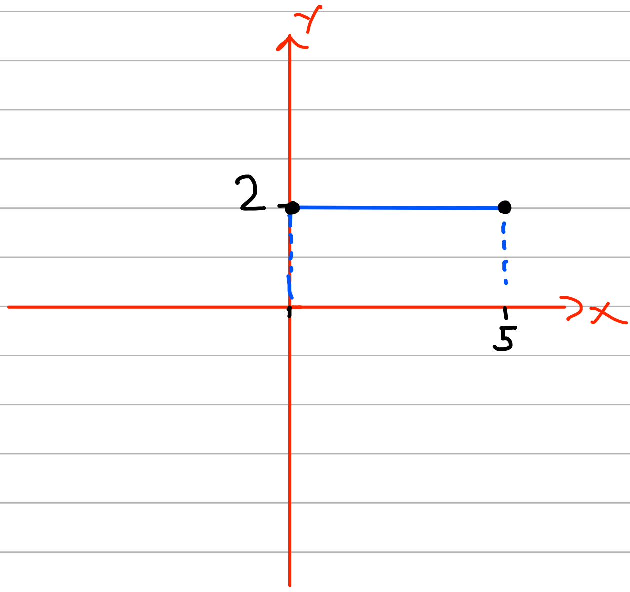
$= \frac{1}{2}(2)(1) - \frac{1}{2}(1)(\frac{1}{2})$

$= 1 - \frac{1}{4} = \frac{4.1 - 1.1}{4} = \frac{3}{4}$

15. (a) $\int_0^5 2\,dx$

$Y = 2$

$A = 2 . 5 = 10$

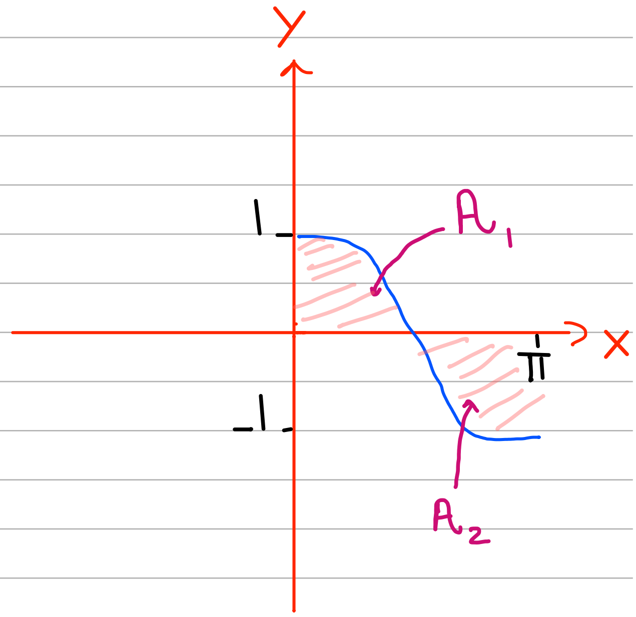


(b) $\int_0^{\pi} \cos x \, dx$

$Y = \cos X$

at $X = 0 \longrightarrow Y = \cos 0 = 1$ $(0, 1)$

at $X = \pi \longrightarrow Y = \cos \pi = -1$ $(\pi, -1)$



A = area of Circle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{4}\pi(r_1^2) - \frac{1}{4}\pi(r_2^2)$

$= \frac{1}{4}\pi(1^2) - \frac{1}{4}\pi(-1^2)$

$= \frac{1}{4}\pi - \frac{1}{4}\pi$

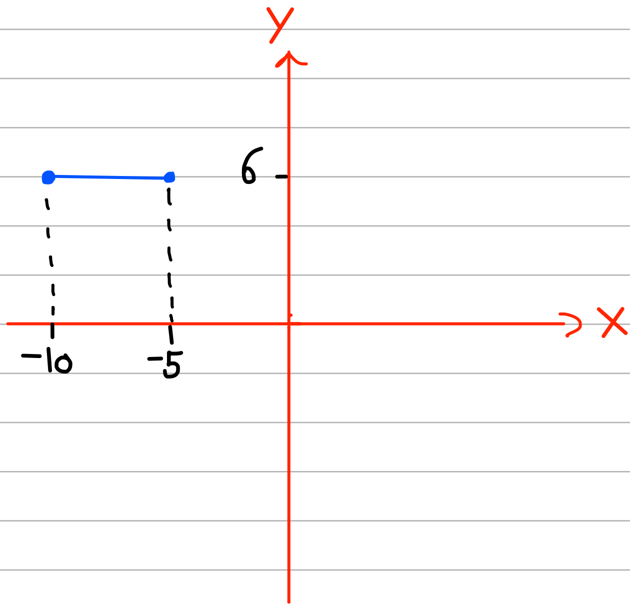
$= 0$

$A_1 = A_2$

16. (a) $\int_{-10}^{-5} 6\,dx$

$Y = 6$

$A = 6 \cdot 5 = 30$



(b) $\int_{-\pi/3}^{\pi/3} \sin x \, dx$

$Y = \sin X$

at $X = \pi/3 \longrightarrow Y = \sin(\pi/3) = \sqrt{3}/2$ $(\pi/3, \sqrt{3}/2)$

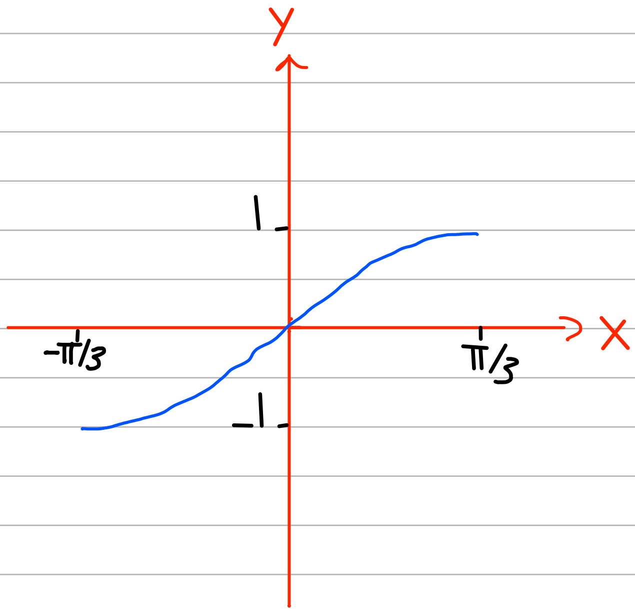
at $X = -\pi/3 \longrightarrow Y = \sin(-\pi/3) = -\sqrt{3}/2$ $(-\pi/3, -\sqrt{3}/2)$

A = area of Circle

$= A_1 + (-A_2)$

$= A_1 - A_2$

$= \frac{1}{4}\pi(r_1^2) - \frac{1}{4}\pi(r_2^2)$



$= \frac{1}{4}\pi(1^2) - \frac{1}{4}\pi(-1^2)$

$= \frac{1}{4}\pi - \frac{1}{4}\pi$

$= 0$

$A_1 = A_2$

17. In each part, evaluate the integral, given that

$$f(x) = \begin{cases} |x-2|, & x \geq 0 \\ x+2, & x < 0 \end{cases}$$

(a) $\int_{-2}^{0} f(x)\, dx$

$Y = X + 2$

at $X = -2 \rightarrow Y = -2 + 2 = 0$ $(-2, 0)$

at $X = 0 \rightarrow Y = 0 + 2 = 2$ $(0, 2)$

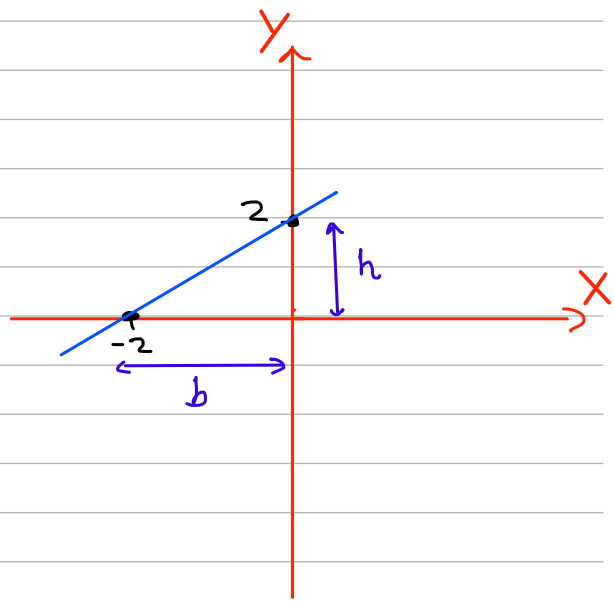
A = area of triangle

$= \frac{1}{2}(\text{base} \cdot \text{height})$

$= \frac{1}{2}(2)(2)$

$= \frac{4}{2}$

$= 2$



(c) $\int_0^6 f(x)\,dx$

$= \int_0^6 |x-2|\,dx$ $\quad (0,2)$

$y = |x-2|$ $\quad (6,4)$

$y=0 \rightarrow 0 = x-2 \rightarrow \boxed{x=2}$

at $x=0 \rightarrow y = |0-2| = |-2| = 2$ $\quad (0,2)$

at $x=6 \rightarrow y = |6-2| = |4| = 4$ $\quad (6,4)$

A = area of triangle

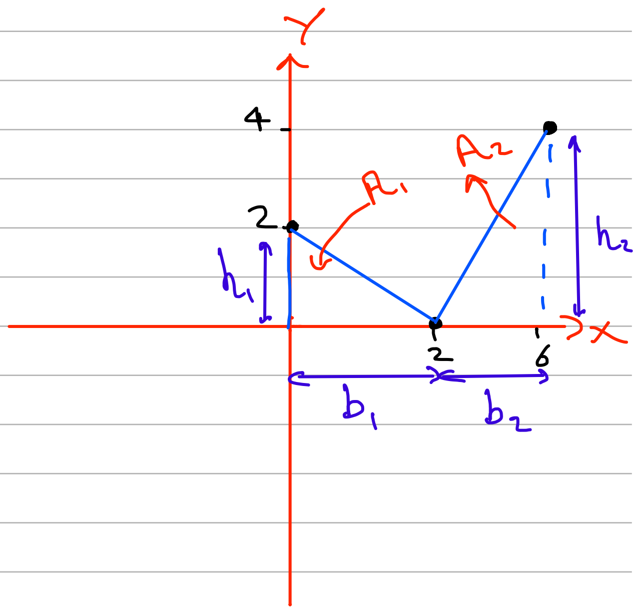
$= A_1 + A_2$

$= \frac{1}{2}(b_1)(h_1) + \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(2) + \frac{1}{2}(4)(4)$

$= 2 + 8$

$= 10$



(d) $\int_{-4}^{6} f(x)\,dx$

$$\int_{-4}^{6} f(X)\,dX = \int_{-4}^{0} f(X)\,dX + \int_{0}^{6} f(X)\,dX$$

$$= \int_{-4}^{0} (X+2)\,dX + \int_{0}^{6} |X-2|\,dX$$

$$= \int_{-4}^{0} (X+2)\,dX + 10$$

$Y = X+2$

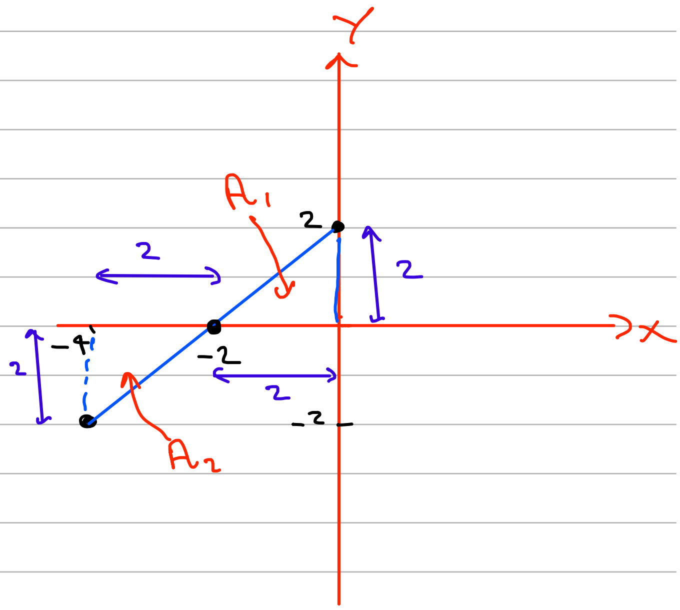
at $X = 0 \rightarrow Y = 0+2 = 2$ $(0, 2)$

at $X = -4 \rightarrow Y = -4+2 = -2$ $(-4, -2)$

A = area of triangle

$= A_1 + (-A_2)$

$= A_1 - A_2$



$= \frac{1}{2}(b_1)(h_1) - \frac{1}{2}(b_2)(h_2)$

$= \frac{1}{2}(2)(2) - \frac{1}{2}(2)(2)$

$= 2 - 2$

$= 0$

$$\int_{-4}^{6} f(X)\,dX = \int_{-4}^{0} (X+2)\,dX + 10$$

$$= 0 + 10 = 10$$

18. In each part, evaluate the integral, given that

$$f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$$

(a) $\int_0^1 f(x)\, dx$

$$\int_0^1 2x\, dX = \frac{2X^2}{2} \Big]_0^1$$

$$= X^2 \Big]_0^1$$

$$= 1^2 - 0^2$$

$$= 1$$

(b) $\int_{-1}^1 f(x)\, dx$

$$\int_{-1}^1 2x\, dX = \frac{\cancel{2}X^2}{\cancel{2}} \Big]_{-1}^1$$

$$= X^2 \Big]_{-1}^1$$

$$= 1^2 - (-1)^2$$

$$= 0$$

(c) $\int_{1}^{10} f(x)\,dx$

$$\int_1^{10} 2\,dx = 2x\Big)_1^{10}$$

$$= 2(10) - 2(1)$$

$$= 20 - 2$$

$$= 18$$

(d) $\int_{1/2}^{5} f(x)\,dx$

$$\int_{1/2}^{5} f(x)dx = \int_{1/2}^{1} f(x)dx + \int_{1}^{5} f(x)dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}}\Bigg)_{1/2}^{1} + (2x)_1^5$$

$$= (x^2)_{1/2}^{1} + (2x)_1^5$$

$$= (1^2 - 1/2^2) + (2.5 - 2.1)$$

$$= \frac{3}{4} + 8$$

$$= \frac{35}{4}$$

21. Find $\int_{-1}^{2} [f(x) + 2g(x)]\,dx$ if

$$\int_{-1}^{2} f(x)\,dx = 5 \quad \text{and} \quad \int_{-1}^{2} g(x)\,dx = -3$$

$= 5 + 2(-3)$

$= 5 - 6$

$= -1$

22. Find $\int_{1}^{4} [3f(x) - g(x)]\,dx$ if

$$\int_{1}^{4} f(x)\,dx = 2 \quad \text{and} \quad \int_{1}^{4} g(x)\,dx = 10$$

$= 3\int_1^4 f(x)\,dx - \int_1^4 g(x)\,dx$

$= 3(2) - 10$

$= 6 - 10$

$= -4$

23. Find $\int_1^5 f(x)\,dx$ if

$$\int_0^1 f(x)\,dx = -2 \quad \text{and} \quad \int_0^5 f(x)\,dx = 1$$

$\int_0^5 f(x)dx = \int_0^1 f(x)dx + \int_1^5 f(x)dx$

$1 = -2 + \int_1^5 f(x)dx$

$1 + 2 = \int_1^5 f(x)dx$

$\int_1^5 f(x)dx = 3$

24. Find $\int_{3}^{-2} f(x)\,dx$ if

$$\int_{-2}^{1} f(x)\,dx = 2 \quad \text{and} \quad \int_{1}^{3} f(x)\,dx = -6$$

$$\int_{3}^{-2} f(x)dx = -\int_{-2}^{3} f(x)dx$$

$$= -\left[\int_{-2}^{1} f(x)dx + \int_{1}^{3} f(x)dx\right]$$

$$= -(2+(-6))$$

$$= -(2-6)$$

$$= 4$$

33–34 Use Theorem 4.5.6 to determine whether the value of the integral is positive or negative. ■

(a) $\int_2^3 \frac{\sqrt{x}}{1-x}\,dx$

$\sqrt{x} \geqslant 0 \longrightarrow x \geqslant 0$

$1-x \longrightarrow x \geqslant 2 \longrightarrow (1-x) < 0$

$f(x) = \frac{\sqrt{x}}{1-x} < 0$ on $[2,3]$

So the integral is negative on $[2,3]$

(b) $\displaystyle\int_0^4 \frac{x^2}{3-\cos x}\,dx$

$x^2 \geqslant 0$ for all x

$-1 \leqslant \cos x \leqslant 1$

$1 \geqslant -\cos x \geqslant -1$

$3+1 \geqslant 3-\cos x \geqslant -1+3$

$4 \geqslant 3-\cos x \geqslant 2$

$\therefore 3-\cos x > 0$ for all x

$f(x) \geqslant 0$ on $[0,4]$

$$\int_0^4 \frac{x^2}{3-\cos x}\,dx \geqslant 0$$

So the integral is Positive

34. (a) $\int_{-3}^{-1} \frac{x^4}{\sqrt{3-x}}\,dx$

$x^4 \geqslant 0 \longrightarrow x \geqslant 0$

$\sqrt{3-x} \geqslant 0$ on $[-1,-3]$

So the integral is Positive on $[-1,-3]$

(b) $\int_{-2}^{2} \frac{x^3-9}{|x|+1}\,dx$

$x^3-9 < 0$ on $[-2,2]$

$|x|+1 > 0$ on $[-2,2]$

So the integral is negative on $[-2,2]$