Section (4.2) :

4.2 THE INDEFINITE INTEGRAL

ANTIDERIVATIVES معكوس الاشتقاق

4.2.1 DEFINITION A function F is called an ***antiderivative*** of a function f on a given open interval if $F'(x) = f(x)$ for all x in the interval.

الدالة F تسمى معكوس المشتقة للدالة f على الفترة المفتوحة اذا كان f $F'(X)=f(x)$ لكل قيم xفي الفترة

$$F(X) = \frac{X^3}{3}$$

$$f(X) = X^2$$

$$F'(X) = \frac{d}{dx}\left(\frac{X^3}{3}\right)$$

$$= \cancel{3} \cdot \frac{X^2}{\cancel{3}}$$

$$= X^2$$

$F(X)$ is antiderivative of $f(X)$

4.2.2 THEOREM *If $F(x)$ is any antiderivative of $f(x)$ on an open interval, then for any constant C the function $F(x)+C$ is also an antiderivative on that interval. Moreover, each antiderivative of $f(x)$ on the interval can be expressed in the form $F(x)+C$ by choosing the constant C appropriately.*

$$F(X) = \frac{1}{3}X^3 + 4$$

$$F'(X) = \frac{d}{dx}\left(\frac{1}{3}X^3 + 4\right)$$

$$= \frac{\cancel{3}}{\cancel{3}}X^2 + 0$$

$$F'(X) = X^2$$

THE INDEFINITE INTEGRAL

The process of finding antiderivatives is called ***antidifferentiation*** or ***integration***. Thus, if

$$\boxed{\frac{d}{dx}[F(x)] = f(x)} \tag{1}$$

then ***integrating*** (or ***antidifferentiating***) the function $f(x)$ produces an antiderivative of the form $F(x) + C$. To emphasize this process, Equation (1) is recast using ***integral notation***,

$$\boxed{\int f(x)\,dx = F(x) + C} \tag{2}$$

where C is understood to represent an arbitrary constant. It is important to note that (1) and (2) are just different notations to express the same fact. For example,

$$\boxed{\int x^2\,dx = \tfrac{1}{3}x^3 + C \quad \text{is equivalent to} \quad \frac{d}{dx}\left[\tfrac{1}{3}x^3\right] = x^2}$$

Note that if we differentiate an antiderivative of $f(x)$, we obtain $f(x)$ back again. Thus,

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x) \tag{3}$$

The expression $\int f(x)\,dx$ is called an ***indefinite integral***. The adjective "indefinite" emphasizes that the result of antidifferentiation is a "generic" function, described only up to a constant term. The "elongated s" that appears on the left side of (2) is called an ***integral sign***,* the function $f(x)$ is called the ***integrand***, and the constant C is called the ***constant of integration***. Equation (2) should be read as:

The integral of $f(x)$ with respect to x is equal to $F(x)$ plus a constant.

INTEGRATION FORMULAS

Table 4.2.1

INTEGRATION FORMULAS

DIFFERENTIATION FORMULA	INTEGRATION FORMULA	DIFFERENTIATION FORMULA	INTEGRATION FORMULA
1. $\dfrac{d}{dx}[x] = 1$	$\displaystyle\int dx = x + C$	5. $\dfrac{d}{dx}[\tan x] = \sec^2 x$	$\displaystyle\int \sec^2 x\,dx = \tan x + C$
2. $\dfrac{d}{dx}\left[\dfrac{x^{r+1}}{r+1}\right] = x^r\ (r \neq -1)$	$\displaystyle\int x^r\,dx = \dfrac{x^{r+1}}{r+1} + C\ (r \neq -1)$	6. $\dfrac{d}{dx}[-\cot x] = \csc^2 x$	$\displaystyle\int \csc^2 x\,dx = -\cot x + C$
3. $\dfrac{d}{dx}[\sin x] = \cos x$	$\displaystyle\int \cos x\,dx = \sin x + C$	7. $\dfrac{d}{dx}[\sec x] = \sec x \tan x$	$\displaystyle\int \sec x \tan x\,dx = \sec x + C$
4. $\dfrac{d}{dx}[-\cos x] = \sin x$	$\displaystyle\int \sin x\,dx = -\cos x + C$	8. $\dfrac{d}{dx}[-\csc x] = \csc x \cot x$	$\displaystyle\int \csc x \cot x\,dx = -\csc x + C$

Example 1 The second integration formula in Table 4.2.1 will be easier to remember if you express it in words:

> *To integrate a power of x (other than −1), add 1 to the exponent and divide by the new exponent.*

Here are some examples:

$$\int x^2\,dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

$$\int x^3\,dx = \frac{x^{3+1}}{3+1} + C = \frac{x^4}{4} + C$$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-5+1}}{-5+1} + C$$

$$= \frac{x^{-4}}{-4} + C = -\frac{1}{4x^4} + C$$

$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{x^{1/2+1}}{1/2+1} + C$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + C = \frac{2}{3}x^{3/2} + C$$

$$= \frac{2}{3}(x^{1/2})^3 + C$$

$$= \frac{2}{3}(\sqrt{x})^3 + C$$

مميزات التكامل غير المحدود

PROPERTIES OF THE INDEFINITE INTEGRAL

Our first properties of antiderivatives follow directly from the simple constant factor, sum, and difference rules for derivatives.

4.2.3 THEOREM *Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that c is a constant. Then:*

(a) *A constant factor can be moved through an integral sign; that is,*

$$\boxed{\int cf(x)\,dx = cF(x) + C}$$

(b) *An antiderivative of a sum is the sum of the antiderivatives; that is,*

$$\boxed{\int [f(x) + g(x)]\,dx = F(x) + G(x) + C}$$

(c) *An antiderivative of a difference is the difference of the antiderivatives; that is,*

$$\boxed{\int [f(x) - g(x)]\,dx = F(x) - G(x) + C}$$

The statements in Theorem 4.2.3 can be summarized by the following formulas:

$$\boxed{\int cf(x)\,dx = c\int f(x)\,dx} \tag{4}$$

$$\boxed{\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx} \tag{5}$$

$$\boxed{\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx} \tag{6}$$

▸ **Example 2** Evaluate

(a) $\int 4 \cos x \, dx$ → نستبعد الثابتة لخارج التكامل

$= 4\int \cos x \, dx$ → تكامل ال Cos وتكاملها يساوي Sin

$= 4 \sin x + C$ → نضيف الثابت C

(b) $\int (x + x^2) \, dx$

$\int x \, dx + \int x^2 \, dx$ → نوزع التكامل

$= \frac{x^2}{2} + \frac{x^3}{3} + C$ → تكامل المتغير x بأن نزود على الاس

واحد و نقسم على الاس الجديد

▸ **Example 3**

$$\int (3x^6 - 2x^2 + 7x + 1)\,dx = 3\int x^6\,dx - 2\int x^2\,dx + 7\int x\,dx + \int 1\,dx$$

$\int 3x^6 dx - \int 2x^2 dx + \int 7x dx + \int dx$ ⟶ نوزع التكامل على جميع الحدود

$3\int x^6 dx - 2\int x^2 dx + 7\int x dx + \int dx$ ⟶ نستبعد الثابت ل خارج التكامل

$3\dfrac{x^{6+1}}{6+1} - 2\dfrac{x^{2+1}}{2+1} + 7\dfrac{x^{1+1}}{1+1} + x + C$ ⟶ نكامل المتغير ب زيادة واحد على الاس و نقسم على الاس الجديد

$3\dfrac{x^7}{7} - 2\dfrac{x^3}{3} + 7\dfrac{x^2}{2} + x + C$

▶ **Example 4** Evaluate

(a) $\int \frac{\cos x}{\sin^2 x}\,dx$

$\int \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}\,dx \longrightarrow$ قمنا بتوزيع المقام على البسط بفك ال sin تربيع ل حاصل ضرب

$= \int \csc x \cot x\,dx \longrightarrow \csc x = \frac{1}{\sin x}, \cot x = \frac{\cos x}{\sin x}$

$= -\csc x + C$

(b) $\int \frac{t^2 - 2t^4}{t^4}\,dt$

$\int \frac{t^2}{t^4} - \frac{2t^4}{t^4}\,dt \longrightarrow$ وزعنا المقام على البسط

$\int \frac{1}{t^2}\,dt - \int 2\,dt$

$\int t^{-2}\,dt - \int 2\,dt \longrightarrow$ نرفع t تربيع للبسط فيتحول الاس لسالب

$= \frac{t^{-2+1}}{-2+1} - 2t + C$

$= \frac{t^{-1}}{-1} - 2t + C$

$= -\frac{1}{t} - 2t + C$