Exercise set (4.2):

Exercise 4.2. P.214-215:

1-2(a)-6-9(c)-10(c)-11-16-25

EXERCISE SET 4.2 Graphing Utility C CAS

1. In each part, confirm that the formula is correct, and state a corresponding integration formula.

(a) $\frac{d}{dx}[\sqrt{1+x^2}] = \frac{x}{\sqrt{1+x^2}}$

(b) $\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2\cos(1+x^3)$

a) $\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{1}{2\sqrt{1+x^2}} \cdot 2x$

$= \frac{x}{\sqrt{1+x^2}}$

$\therefore$ formula is Correct

$\int \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2} + C$

b) $\frac{d}{dx}\left(\frac{1}{3}\sin(1+x^3)\right) = \frac{1}{3}\cos(1+x^3).(3x^2)$

$= x^2\cos(1+x^3)$

$\therefore$ formula is Correct

$\int x^2\cos(1+x^3)\,dx = \frac{1}{3}\sin(1+x^3) + C$

2. In each part, confirm that the stated formula is correct by differentiating.

(a) $\int x \sin x \, dx = \sin x - x \cos x + C$

$\frac{d}{dx}(\sin x - x\cos x + C) =$

$= \cos x - \left(\frac{d}{dx}(x).(\cos x) + (x).\frac{d}{dx}(\cos x)\right)$

$= \cos x - \cos x + x(-\sin x)$

$= \cos x - \cos x + x\sin x$

$= x\sin x$

$\therefore$ formula is Correct

5–8 Find the derivative and state a corresponding integration formula. ■

5. $\frac{d}{dx}[\sqrt{x^3+5}]$

$= \frac{3x^2}{2\sqrt{x^3+5}}$

$\int \frac{3x^2}{2\sqrt{x^3+5}} dx = \sqrt{x^3+5} + C$

6. $\frac{d}{dx}\left[\frac{x}{x^2+3}\right]$

$$= \frac{(x^2+3)\cdot\frac{d}{dx}(x) - (x)\cdot\frac{d}{dx}(x^2+3)}{(x^2+3)^2}$$

$$= \frac{(x^2+3)\cdot(1) - (x)(2x)}{(x^2+3)^2}$$

$$= \frac{(x^2+3)-(2x^2)}{(x^2+3)^2}$$

$$= \frac{-x^2+3}{(x^2+3)^2}$$

$$= \frac{3-x^2}{(x^2+3)^2}$$

$$\int \frac{3-x^2}{(x^2+3)^2}\,dx = \frac{x}{x^2+3} + C$$

8. $\dfrac{d}{dx}[\sin x - x\cos x]$

$= \cos x - 1.\cos x - x(-\sin x)$

$= \cancel{\cos x} - \cancel{\cos x} + x\sin x$

$= x\sin x$

$\int x\sin x\,dx = \sin x - x\cos x + C$

9–10 Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule (Formula 2 in Table 4.2.1). ■

(a) $\int x^8\,dx$

$= \frac{x^9}{9} + C$

(b) $\int x^{5/7}\,dx$

$= \frac{x^{12/7}}{12/7} + C$

$= \frac{7}{12}\,x^{12/7} + C$

(c) $\int x^3\sqrt{x}\,dx$

$\int x^3 x^{1/2}\,dx = \int x^{3+\frac{1}{2}}\,dx$

$= \int x^{7/2}\,dx$

$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} + C$

$= \frac{x^{9/2}}{9/2} + C = \frac{2}{9}\,x^{9/2} + C$

10. (a) $\int \sqrt[3]{x^2}\,dx$

$= \int (x^{1/3})^2\, dx$

$= \int x^{2/3}\, dx$

$= \dfrac{x^{5/3}}{5/3} + C$

$= \dfrac{3}{5} x^{5/3} + C$

(b) $\int \dfrac{1}{x^6}\,dx$

$= \int x^{-6}\, dx$

$= -\dfrac{x^{-5}}{5} + C$

(c) $\int x^{-7/8}\,dx$

$= \dfrac{x^{-7/8+1}}{-7/8+1} + C$

$= \dfrac{x^{1/8}}{1/8} + C$

$= 8x^{1/8} + C$

$= 8\sqrt[8]{x} + C$

11–14 Evaluate each integral by applying Theorem 4.2.3 and Formula 2 in Table 4.2.1 appropriately. ■

11. $\int \left[5x + \frac{2}{3x^5}\right] dx$

$$\int 5x\,dx + \int \frac{2}{3x^5}\,dx$$

$$5\int x\,dx + \frac{2}{3}\int \frac{1}{x^5}\,dx$$

$$5\int x\,dx + \frac{2}{3}\int x^{-5}\,dx$$

$$5\frac{x^{1+1}}{1+1} + \frac{2}{3}\left(\frac{x^{-5+1}}{-5+1}\right) + C$$

$$5\frac{x^2}{2} + \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + C$$

$$\frac{5x^2}{2} - \frac{2x^{-4}}{12} + C$$

$$\frac{5x^2}{2} - \frac{x^{-4}}{6} + C$$

$$\frac{5x^2}{2} - \frac{1}{6x^4} + C$$

12. $\int [x^{-1/2} - 3x^{7/5} + \frac{1}{9}]\, dx$

$\int x^{-1/2} dx - 3\int x^{7/5} dx + \int \frac{1}{9} dx$

$= \frac{x^{1/2}}{1/2} - 3\frac{x^{12/5}}{12/5} + \frac{1}{9}x + C$

$= 2x^{1/2} - 3\frac{5}{12}x^{12/5} + \frac{1}{9}x + C$

$= 2x^{1/2} - \frac{5}{4}x^{12/5} + \frac{1}{9}x + C$

13. $\int [x^{-3} - 3x^{1/4} + 8x^2]\, dx$

$= \frac{x^{-2}}{-2} - 3\frac{x^{5/4}}{5/4} + 8\frac{x^3}{3} + C$

$= -\frac{x^{-2}}{2} - \frac{12}{5}x^{5/4} + 8\frac{x^3}{3} + C$

14. $\int \left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}}\right] dy$

$= 10\int y^{-3/4}\,dy - \int y^{1/3}\,dy + 4\int y^{-1/2}\,dy$

$= 10\,\frac{y^{1/4}}{1/4} - \frac{y^{4/3}}{4/3} + 4\,\frac{y^{1/2}}{1/2} + C$

$= 40\,y^{1/4} - \frac{3}{4}\,y^{4/3} + 8\,y^{1/2} + C$

$= 40\sqrt[4]{y} - \frac{3}{4}\sqrt[3]{y^4} + 8\sqrt{y} + C$

15–30 Evaluate the integral and check your answer by differentiating.

15. $\int x(1+x^3)\,dx$

$= \int X + X^4 \, dX$

$= \dfrac{X^2}{2} + \dfrac{X^5}{5} + C$

_ Check the answer by differentiating

$\dfrac{d}{dX}\left(\dfrac{X^2}{2} + \dfrac{X^5}{5} + C\right)$

$= \dfrac{\cancel{2}X}{\cancel{2}} + \dfrac{5X^4}{5}$

$= X + X^4$

16. $\int (2 + y^2)^2 \, dy$

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

$$\int (4 + 4y^2 + y^4) \, dy$$

$$\int 4 \, dy + 4 \int y^2 \, dy + \int y^4 \, dy$$

$$= 4y + 4 \frac{y^3}{3} + \frac{y^5}{5}$$

_ Check the answer by differentiating

$$\frac{d}{dy}\left(4y + 4\frac{y^3}{3} + \frac{y^5}{5}\right)$$

$$= 4 + \cancel{3}\left(\frac{4}{\cancel{3}}\right) y^{3-1} + \cancel{5}\left(\frac{1}{\cancel{5}}\right) y^{5-1}$$

$$= 4 + 4y^2 + y^4$$

$$= (2 + y^2)^2$$

17. $\int x^{1/3}(2-x)^2\,dx$

$$= \int x^{1/3}(4-4x+x^2)\,dx$$

$$= \int (4x^{1/3} - 4x^{4/3} + x^{7/3})\,dx$$

$$= 4\frac{x^{4/3}}{4/3} - 4\frac{x^{7/3}}{7/3} + \frac{x^{10/3}}{10/3} + C$$

$$= 3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$$

- Check the answer by differentiating

$$\frac{d}{dx}\left(3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C\right)$$

$$= 4x^{1/3} - 4x^{4/3} + x^{7/3}$$

$$= x^{1/3}(4-4x+x^2)$$

$$= x^{1/3}(2-x)^2$$

18. $\int (1 + x^2)(2 - x)\, dx$

$$\int (2 - x + 2x^2 - x^3)\, dx$$

$$= 2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C$$

_ Check the answer by differentiating

$$\frac{d}{dx}\left(2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C\right)$$

$$= 2 - x + 2x^2 - x^3$$

19. $\displaystyle\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx$

$\int x^{-4}(x^5 + 2x^2 - 1)\,dx$

$\int (x + 2x^{-2} - x^{-4})\,dx$

$= \dfrac{x^2}{2} + 2\,\dfrac{x^{-1}}{-1} - \dfrac{x^{-3}}{-3} + C$

$= \dfrac{x^2}{2} - \dfrac{2}{x^1} + \dfrac{1}{3x^3} + C$

_ Check the answer by differentiating

$\dfrac{d}{dx}\left(\dfrac{x^2}{2} - \dfrac{2}{x^1} + \dfrac{1}{3x^3} + C\right)$

$= x + 2x^{-2} - x^{-4})$

20. $\int \frac{1-2t^3}{t^3}\,dt$

$\int t^{-3}(1-2t^3)\,dt$

$\int (t^{-3}-2)\,dt$

$= \frac{t^{-2}}{-2} - 2t + C$

$= \frac{1}{2t^2} - 2t + C$

- Check the answer by differentiating

$\frac{d}{dt}\left(\frac{1}{2t^2} - 2t + C\right)$

$= t^{-3} - 2$

23. $\int \sec x(\sec x + \tan x)\, dx$

$\int (\sec^2 x + \sec x \tan x) dx$

$= \tan x + \sec x + C$

24. $\int \csc x(\sin x + \cot x)\, dx$

$\int (\csc x \sin x + \csc x \cot x) dx$

$\int (\frac{1}{\sin x} \cdot \sin x + \csc x \cot x) dx$

$\int (1 + \csc x \cot x) dx$

$= x - \csc x + C$

25. $\displaystyle\int \frac{\sec\theta}{\cos\theta}\,d\theta$

$\boxed{\frac{1}{\cos\theta} = \sec\theta}$

$\int \sec\theta \cdot \frac{1}{\cos\theta}\,d\theta$

$\int \sec\theta . \sec\theta\,d\theta$

$\int \sec^2\theta\,d\theta$

$= \tan\theta + C$

_ Check the answer by differentiating

$\frac{d}{d\theta}(\tan\theta + C)$

$= \sec^2\theta$

$= \sec\theta . \sec\theta$

$= \sec\theta \cdot \frac{1}{\cos\theta}$

$= \frac{\sec\theta}{\cos\theta}$

26. $\int \frac{dy}{\csc y}$

$= \int \frac{1}{\csc y} dy$

$= \int \sin y \, dy$

$= \cos y + C$

27. $\int \frac{\sin x}{\cos^2 x} dx$

$= \int \frac{\sin x}{\cos x \cos x} dx$

$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$

$= \int \tan x \sec x \, dx$

$= \sec x + C$

29. $\int [1 + \sin^2\theta \csc\theta]\, d\theta$

$$= \int \left(1 + \sin^{\cancel{2}}\theta \cdot \frac{1}{\cancel{\sin\theta}} \right) d\theta$$

$$= \int \left(1 + \sin\theta \right) d\theta$$

$$= \theta - \cos\theta + C$$

30. $\int \frac{\sec x + \cos x}{2\cos x}\, dx$

$$= \frac{1}{2} \int \left(\frac{\sec x}{\cos x} + \frac{\cos x}{\cos x} \right) dx$$

$$= \frac{1}{2} \int \left(\frac{\frac{1}{\cos x}}{\cos x} + 1 \right) dx$$

$$= \frac{1}{2} \int \left(\frac{1}{\cos^2 x} + 1 \right) dx$$

$$= \frac{1}{2} \tan x + x + C$$

33–36 True–False Determine whether the statement is true or false. Explain your answer.

33. If $F(x)$ is an antiderivative of $f(x)$, then (true)

$$\int f(x)\,dx = F(x) + C$$