IQ ACADEMY

Exercise set (4.9):

Exercise 4.9 P.270-271:

1(b-c)-2(b-c)-9-13-14-25-28-33-37

1–4 Express the integral in terms of the variable u, but do not evaluate it.

1. (a) $\int_1^3 (2x-1)^3\,dx;\ u = 2x-1$

$du = 2\,dX \longrightarrow \frac{du}{2} = dX$

at $X = 1 \longrightarrow u = 2(1) - 1 = 1$

at $X = 3 \longrightarrow u = 2(3) - 1 = 5$

$= \frac{1}{2}\int_1^5 u^3\,du$

$= \frac{1}{2}\cdot\left(\frac{u^4}{4}\right)\Big|_1^5$

(b) $\int_0^4 3x\sqrt{25-x^2}\,dx;\ u = 25 - x^2$

$u = 25 - x^2$

$du = -2x\,dx \longrightarrow -\frac{du}{2} = x\,dx$

at $x = 0 \longrightarrow u = 25 - 0^2 = 25$

at $x = 4 \longrightarrow u = 25 - 4^2 = 9$

$$\int_9^{25} 3\sqrt{u}\left(-\frac{du}{2}\right)$$

$$= -\frac{3}{2}\int_9^{25} 3\sqrt{u}\,du$$

(c) $\int_{-1/2}^{1/2} \cos(\pi\theta)\, d\theta;\ u = \pi\theta$

$u = \pi\theta$

$du = \pi\, d\theta \longrightarrow \frac{du}{\pi} = d\theta$

at $\theta = -\frac{1}{2} \longrightarrow u = -\frac{\pi}{2}$

at $\theta = \frac{1}{2} \longrightarrow u = \frac{\pi}{2}$

$$\int_{-\pi/2}^{\pi/2} \cos(u) \frac{du}{\pi}$$

$$\frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(u)\, du$$

2. (a) $\int_{-1}^{4} (5-2x)^8\, dx;\ u = 5-2x$

$du = -2\,dx \longrightarrow -\dfrac{du}{2} = dx$

at $x = -1 \longrightarrow u = 5 - 2(-1) = 7$

at $x = 4 \longrightarrow u = 5 - 2(4) = -3$

$$= -\frac{1}{2}\int_{7}^{-3} u^8\, du$$

$$= \frac{1}{2}\int_{-3}^{7} u^8\, du$$

(b) $\displaystyle\int_{-\pi/3}^{2\pi/3} \frac{\sin x}{\sqrt{2+\cos x}}\,dx;\ u = 2+\cos x$

$du = 0 - \sin x\,dx$

at $X = \frac{2\pi}{3} \longrightarrow u = 2 + \cos\left(\frac{2\pi}{3}\right) = 2 + \left(-\frac{1}{2}\right) = \frac{3}{2}$

at $X = -\frac{\pi}{3} \longrightarrow u = 2 + \cos\left(-\frac{\pi}{3}\right) = 2 + \frac{1}{2} = \frac{5}{2}$

$$\int_{3/2}^{5/2} \frac{1}{\sqrt{u}}\,du$$

(c) $\int_0^{\pi/4} \tan^2 x \sec^2 x \, dx; \; u = \tan x$

$du = \sec^2 x dx$

at $X = 0 \longrightarrow u = \tan 0 = 0$

at $X = \frac{\pi}{4} \longrightarrow u = \tan\left(\frac{\pi}{4}\right) = 1$

$\int_0^1 u^2 du$

3–12 Evaluate the definite integral two ways: first by a u-substitution in the definite integral and then by a u-substitution in the corresponding indefinite integral.

3. $\int_0^1 (2x+1)^3\,dx$

$u = 2X + 1$

$du = 2dX \longrightarrow \frac{du}{2} = dX$

at $X = 0 \longrightarrow u = 2.0 + 1 = 1$

at $X = 1 \longrightarrow u = 2.1 + 1 = 3$

$\frac{1}{2}\int_1^3 u^3\,du$

$= \frac{1}{2}\cdot\left(\frac{u^4}{4}\right)_1^3$

$= \frac{1}{8}(u^4)_1^3$

$= \frac{1}{8}(3^4 - 1^4)$

$= \frac{1}{8}(81 - 1) = 10$

– second method:

$\frac{1}{2}\int_0^1 u^3\,du$

$= \frac{1}{2}\cdot\left(\frac{u^4}{4}\right)_0^1$

$$= \frac{1}{8}\left(u^4\right)_0^1$$

$$= \frac{1}{8}\left(\left(2x+1\right)^4\right)_0^1$$

$$= \frac{1}{8}\left(\left(2.1+1\right)^4 - \left(2.0+1\right)^4\right)$$

$$= \frac{1}{8}\left(\left(3\right)^4 - \left(1\right)\right)$$

$$= \frac{1}{8}\left(81 - 1\right) = 10$$

4. $\int_1^2 (4x-2)^3\,dx$

$w = 4x - 2$

$dw = 4dx \rightarrow \frac{dw}{4} = dx$

at $x = 1 \rightarrow w = 4.1 - 2 = 2$

at $x = 2 \rightarrow w = 4.2 - 2 = 6$

$\frac{1}{4}\int_2^6 w^3\,dw$

$= \frac{1}{4}\cdot\left(\frac{w^4}{4}\right)_2^6$

$= \frac{1}{16}(w^4)_2^6$

$= \frac{1}{16}(6^4 - 2^4)$

$= \frac{1}{16}(1296 - 16) = 80$

- second method:

$\frac{1}{4}\int_1^2 w^3\,dw$

$= \frac{1}{4}\cdot\left(\frac{w^4}{4}\right)_1^2$

$= \frac{1}{16}(w^4)_1^2$

$$= \frac{1}{16}(4x-2)^4 \Big)_{1}^{2}$$

$$= \frac{1}{16}\left((4 \cdot 2 - 2)^4 - (4 \cdot 1 - 2)^4\right)$$

$$= \frac{1}{16}(1296 - 16) = 80$$

5. $\int_0^1 (2x-1)^3\,dx$

$u = 2x - 1$

$du = 2dx \longrightarrow \frac{du}{2} = dx$

at $x = 0 \longrightarrow u = 2 \cdot 0 - 1 = -1$

at $x = 1 \longrightarrow u = 2 \cdot 1 - 1 = 1$

$\frac{1}{2}\int_{-1}^{1} u^3\,du$

$= \frac{1}{2} \cdot \left(\frac{u^4}{4}\right)_{-1}^{1}$

$= \frac{1}{8}(u^4)_{-1}^{1}$

$= \frac{1}{8}(1^4 - (-1)^4)$

$= \frac{1}{8}(1 - 1) = 0$

- second method:

$\frac{1}{2}\int_{0}^{1} u^3\,du$

$= \frac{1}{2} \cdot \left(\frac{u^4}{4}\right)_{0}^{1}$

$$= \frac{1}{8}\left((2x-1)^4 - (2x-1)^4\right)\Big|_0^1$$

$$= \frac{1}{8}\left((2.1-1)^4 - (2.0-1)^4\right)$$

$$= \frac{1}{8}(1-1) = 0$$

6. $\int_1^2 (4-3x)^8\,dx$

$u = 4 - 3x$

$du = -3\,dx \longrightarrow -\frac{du}{3} = dx$

at $x = 1 \longrightarrow u = 4 - 3.1 = 1$

at $x = 2 \longrightarrow u = 4 - 3.2 = -2$

$-\frac{1}{3}\int_1^{-2} u^8\,du = \frac{1}{3}\int_{-2}^{1} u^8\,du$

$= \frac{1}{3}\cdot\left(\frac{u^9}{9}\right)_{-2}^{1}$

$= \frac{1}{27}\left(u^9\right)_{-2}^{1}$

$= \frac{1}{27}\left(1^9 - (-2)^9\right)$

$= \frac{1}{27}\left(1 - (-512)\right) = 19$

- second method:

$-\frac{1}{3}\int_1^{2} u^8\,du$

$= -\frac{1}{3}\cdot\left(\frac{u^9}{9}\right)_1^2$

$$= -\frac{1}{27}\left(u^9\right)\Big|_1^2$$

$$= -\frac{1}{27}\left((4-3x)^9\right)\Big|_1^2$$

$$= -\frac{1}{27}\left((4-3\cdot 2)^9 - (4-3\cdot 1)^9\right)$$

$$= -\frac{1}{27}\left((-2)^9 - (1)^9\right)$$

$$= -\frac{1}{27}\left(-512-1\right) = 19$$

9. $\int_0^{\pi/2} 4\sin(x/2)\,dx$

$u = X/2$

$du = \frac{1}{2}dX \longrightarrow 2du = dX$

at $X = 0 \longrightarrow u = 0/2 = 0$

at $X = \frac{\pi}{2} \longrightarrow u = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

$8\int_0^{\pi/4} \sin(u)\,du$

$= 8\left(-\cos(u)\right)_0^{\pi/4}$

$= 8\left(-\cos\left(\frac{\pi}{4}\right) - \left(-\cos(0)\right)\right)$

$= 8\left(-\frac{\sqrt{2}}{2} + 1\right)$

$= -4\sqrt{2} + 8$

10. $\int_0^{\pi/6} 2\cos 3x \, dx$

$u = 3X$

$du = 3\,dX \longrightarrow \frac{du}{3} = dX$

at $X = 0 \longrightarrow u = 0$

at $X = \frac{\pi}{6} \longrightarrow u = \frac{3\pi}{6}$

$\frac{2}{3}\int_0^{3\pi/6} \cos(u)\,du$

$= \frac{2}{3}(\sin u)_0^{3\pi/6}$

$= \frac{2}{3}\left(\sin\left(\frac{3\pi}{6}\right) - \sin(0)\right)$

$= \frac{2}{3}\left(\sin\left(\frac{\pi}{2}\right) - 0\right)$

$= \frac{2}{3}(1) = \frac{2}{3}$

- second method:

$\frac{2}{3}\int_0^{\pi/6} \cos(u)\,du$

$\frac{2}{3}\left(\sin(u)\right)_0^{\pi/6}$

$$= \frac{2}{3}\left(\sin(3x)\right)_0^{\pi/6}$$

$$= \frac{2}{3}\left(\sin\left(\frac{3\pi}{6}\right) - \sin(0)\right)$$

$$= \frac{2}{3}\left(\sin\left(\frac{\pi}{2}\right) - 0\right)$$

$$= \frac{2}{3}(1) = \frac{2}{3}$$

11. $\int_{-2}^{-1} \frac{x}{(x^2+2)^3}\,dx$

$u = x^2 + 2$

$du = 2x\,dx \longrightarrow \frac{du}{2} = x\,dx$

at $x = -2 \longrightarrow u = -2^2 + 2 = 6$

at $x = -1 \longrightarrow u = -1^2 + 2 = 3$

$$\frac{1}{2}\int_6^3 \frac{1}{u^3}\,du$$

$$= -\frac{1}{2}\int_6^3 \frac{1}{u^3}\,du$$

$$= -\frac{1}{2}\left(-\frac{1}{2u^2}\right)_6^3$$

$$= \frac{1}{2}\left(\frac{1}{2u^2}\right)_6^3$$

$$= \frac{1}{2}\left(\frac{1}{2.6^2} - \frac{1}{2.3^2}\right)$$

$$= \frac{1}{2} \cdot -\frac{1}{24} = -\frac{1}{48}$$

- second method:

$$\frac{1}{2}\int_{-2}^{-1} \frac{1}{u^3}\,du$$

$$= \frac{1}{2}\left(-\frac{1}{2u^2}\right)_{-2}^{-1}$$

$$= -\frac{1}{2}\left(\frac{1}{2u^2}\right)_{-2}^{-1}$$

$$= -\frac{1}{2}\left(\frac{1}{2(x^2+2)^2}\right)_{-2}^{-1}$$

$$= -\frac{1}{4}\left(\frac{1}{(-1^2+2)^2} - \frac{1}{(-2^2+2)^2}\right)$$

$$= -\frac{1}{4}\left(\frac{1}{9} - \frac{1}{36}\right)$$

$$= -\frac{1}{48}$$

13–16 Evaluate the definite integral by expressing it in terms of u and evaluating the resulting integral using a formula from geometry.

13. $\int_{-5/3}^{5/3} \sqrt{25 - 9x^2}\, dx;\ u = 3x$

$u = 3x$

$du = 3\,dx \longrightarrow \frac{du}{3} = dx$

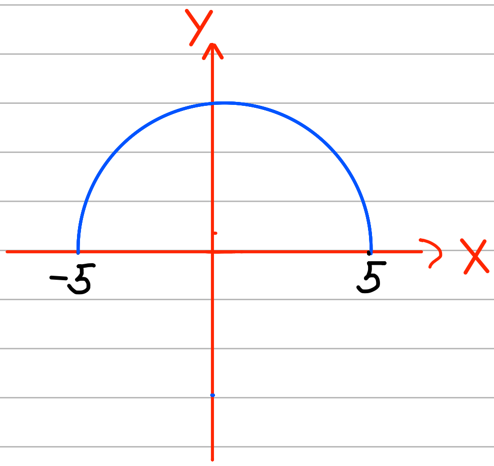
at $x = -\frac{5}{3} \longrightarrow u = \cancel{3} \cdot -\frac{5}{\cancel{3}} = -5$

at $x = \frac{5}{3} \longrightarrow u = \cancel{3} \cdot \frac{5}{\cancel{3}} = 5$

$\frac{1}{3}\int_{-5}^{5} \sqrt{25-u^2}$

$y = \sqrt{25-u^2} \longrightarrow y^2 = (\cancel{\sqrt{25-u^2}})^{\cancel{2}}$

$y^2 = 25 - u^2 \longrightarrow y^2 + u^2 = 25$



A = area of circle

$= \frac{1}{2}\pi r^2 = \frac{1}{3}\left(\frac{1}{2}\pi r^2\right)$

$= \frac{25\pi}{6}$

14. $\displaystyle\int_0^2 x\sqrt{16-x^4}\,dx;\ u = x^2$

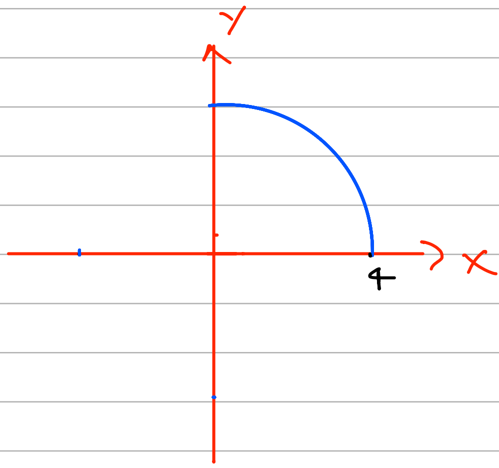
$u = x^2$

$du = 2x\,dx \longrightarrow \dfrac{du}{2} = x\,dx$

at $x = 0 \longrightarrow u = 0^2 = 0$

at $x = 2 \longrightarrow u = 2^2 = 4$

$$\int_0^4 \sqrt{16-u^2}\,\frac{du}{2} = \frac{1}{2}\int_0^4 \sqrt{16-u^2}\,du$$



$Y = \sqrt{16-u^2}$

$Y^2 + u^2 = 16$

A = area of circle

$$= \frac{1}{4}\pi r^2 = \frac{1}{2}\left(\frac{1}{4}\pi r^2\right)$$

$$= \frac{1}{2}\left(\frac{1}{4}\pi (4)^2\right) = 2\pi$$

21–34 Evaluate the integrals by any method.

21. $\int_1^5 \frac{dx}{\sqrt{2x-1}}$

$u = 2x - 1$

$du = 2dx \longrightarrow \frac{du}{2} = dx$

$\frac{1}{2}\int \frac{1}{\sqrt{u}} = \frac{1}{\cancel{2}} \cdot \cancel{2}\sqrt{u}$

$= \sqrt{u}$

$= \left(\sqrt{2x-1}\right)_1^5$

$= \left(\sqrt{2(5)-1} - \sqrt{2(1)-1}\right)$

$= \sqrt{9} - \sqrt{1} = 3 - 1 = 2$

22. $\int_1^2 \sqrt{5x-1}\,dx$

$u = 5x - 1$

$du = 5\,dx \longrightarrow \frac{du}{5} = dx$

$\frac{1}{5}\int \sqrt{u}\,du$

$= \frac{1}{5} \cdot \frac{u^{1/2}}{1/2}$

$= \frac{2}{5} \cdot u^{1/2}$

$= \frac{2}{5}\left((5x-1)^{1/2}\right)\Big|_1^2$

$= \frac{2}{5}\left((5(2)-1)^{1/2} - (5(1)-1)^{1/2}\right)$

$= \frac{2}{5}\left(9^{1/2} - 4^{1/2}\right) = \frac{38}{15}$

23. $\displaystyle\int_{-1}^{1} \frac{x^2\,dx}{\sqrt{x^3+9}}$

$u = x^3 + 9$

$du = 3x^2\,dx \rightarrow \frac{du}{3} = x^2\,dx$

$$\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} = \frac{2}{3}\,u^{1/2}$$

$$= \frac{2}{3}\left((x^3+9)^{1/2}\right)_{-1}^{1}$$

$$= \frac{2}{3}\left((1^3+9)^{1/2} - (-1^3+9)^{1/2}\right)$$

$$= \frac{2\sqrt{10} - 4\sqrt{2}}{3}$$

24. $\int_{\pi/2}^{\pi} 6 \sin x \,(\cos x + 1)^5 \, dx$

$6\int \sin x(\cos x+1)^5 dx$

$u = \cos(x+1)$

$du = -\sin x\, dx \rightarrow -du = \sin x\, dx$

$-6\int u^5 du$

$= -6 \dfrac{u^6}{6}$

$= -u^6$

$= -(\cos(x+1))\Big|_{\pi/2}^{\pi}$

$= -\left((\cos(\pi+1))^6 - (\cos(\tfrac{\pi}{2}+1))^6\right)$

$= 1$

25. $\int_1^3 \frac{x+2}{\sqrt{x^2+4x+7}}\,dx$

$u = x^2+4x+7$

$du = 2x+4\,dx \longrightarrow \frac{du}{2} = x+2\,dx$

at $x = 1 \longrightarrow u = 1^2+4(1)+7 = 12$

at $x = 3 \longrightarrow u = 3^2+4(3)+7 = 28$

$\int_{12}^{28} \frac{1}{\sqrt{u}}\frac{du}{2} = \frac{1}{2}\int_{12}^{28} u^{-1/2}\,du$

$= \frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right)_{12}^{28} = \frac{1}{\cancel{2}}\cdot\cancel{2}\left(u^{1/2}\right)_{12}^{28}$

$= \left(u^{1/2}\right)_{12}^{28}$

$= \left(\sqrt{28} - \sqrt{12}\right)$

$= \left(\sqrt{4\cdot7}\right) - \left(\sqrt{4\cdot3}\right)$

$= 2\sqrt{7} - 2\sqrt{3}$

28. $\displaystyle\int_0^{\pi/4} \sqrt{\tan x}\,\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

at $x = 0 \longrightarrow u = \tan(0) = 0$

at $x = \pi/4 \longrightarrow u = \tan(\pi/4) = 1$

$\int_0^1 \sqrt{u}\,du = \int_0^1 (u)^{1/2}\,du$

$= \left(\dfrac{u^{3/2}}{3/2}\right)\Big|_0^1 = \dfrac{2}{3}\left(u^{3/2}\right)\Big|_0^1$

$= \dfrac{2}{3}\left((1)^{3/2} - (0)^{3/2}\right)$

$= \dfrac{2}{3}$

33. $\int_0^1 \frac{y^2\,dy}{\sqrt{4-3y}}$

$u = 4 - 3Y$

$du = -3dY \longrightarrow -\frac{du}{3} = dY$

$u = 4 - 3Y \longrightarrow Y = -\frac{u-4}{3}$

at $Y = 0 \longrightarrow u = 4 - 3(0) = 4$

at $Y = 1 \longrightarrow u = 4 - 3(1) = 1$

$$-\frac{1}{3}\int_4^1 \frac{\left(-\frac{u-4}{3}\right)^2}{\sqrt{u}}\,du$$

$$-\frac{1}{3}\int_4^1 \frac{\frac{(u-4)^2}{9}}{\sqrt{u}}\,du = -\frac{1}{3}\int_4^1 \frac{(u-4)^2}{9\sqrt{u}}\,du$$

$$= -\frac{1}{27}\int_4^1 \frac{(u-4)^2}{(u)^{1/2}}\,du = -\frac{1}{27}\int_4^1 \frac{u^2-8u+16}{(u)^{1/2}}\,du$$

$$= \frac{1}{27}\int_1^4 u^{3/2} - 8u^{1/2} + 16u^{-1/2}\,du$$

$$= \frac{1}{27}\left(\frac{u^{5/2}}{5/2} - 8\,\frac{u^{3/2}}{3/2} - 16\,\frac{u^{1/2}}{1/2}\right)_1^4$$

$$= \frac{1}{27}\left[\frac{2}{5}u^{5/2} - 8.\frac{2}{3}u^{3/2} - 16.2\,u^{1/2}\right]_1^4$$

$$= \frac{1}{27}\left[\frac{2}{5}u^{5/2} - \frac{16}{3}u^{3/2} - 32\,u^{1/2}\right]_1^4$$

$$= \frac{1}{27}\left(\left(\frac{2}{5}(4)^{5/2} - \frac{16}{3}(4)^{3/2} - 32(4)^{1/2}\right)\right.$$

$$\left. - \left(\frac{2}{5}(1)^{5/2} - \frac{16}{3}(1)^{3/2} - 32(1)^{1/2}\right)\right)$$

$$= \frac{1}{27}\left(\left(\frac{64}{5} - \frac{128}{3} - 64\right) - \left(-\frac{554}{15}\right)\right)$$

$$= \frac{1}{27}\left(\left(-\frac{1408}{15}\right) - \left(-\frac{554}{15}\right)\right)$$

37. (a) Find $\int_0^1 f(3x+1)\,dx$ if $\int_1^4 f(x)\,dx = 5$.

(b) Find $\int_0^3 f(3x)\,dx$ if $\int_0^9 f(x)\,dx = 5$.

(c) Find $\int_{-2}^0 xf(x^2)\,dx$ if $\int_0^4 f(x)\,dx = 1$.

a) $u = 3x+1$

$du = 3\,dx \longrightarrow \frac{du}{3} = dx$

at $x = 0 \longrightarrow u = 3.0 + 1 = 1$

at $x = 1 \longrightarrow u = 3.1 + 1 = 4$

$\int_1^4 f(u)\frac{du}{3} = \frac{1}{3}\int_1^4 f(u)\,du$

$= \frac{1}{3}.5 = \frac{5}{3}$

b) $u = 3x$

$du = 3\,dx \longrightarrow \frac{du}{3} = dx$

at $x = 0 \longrightarrow u = 3.0 = 0$

at $x = 3 \longrightarrow u = 3.3 = 9$

$\int_0^9 f(u)\frac{du}{3} = \frac{1}{3}\int_0^9 f(u)\,du$

$= \frac{1}{3}.5 = \frac{5}{3}$

C) $u = 2x$

$du = 2x\,dx \longrightarrow \frac{du}{2} = x\,dx$

at $x = -2 \longrightarrow u = -2^2 = 4$

at $x = 0 \longrightarrow u = 0^2 = 0$

$$\frac{1}{2}\int_4^0 f(u)\,du = -\frac{1}{2}\int_0^4 f(u)\,du$$

$$= -\frac{1}{2}(1)$$

$$= -\frac{1}{2}$$